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EXPERIMENTS ON THE HYDRODYNAMIC STABILITY OF HELIUM II BETWEEN ROTATING CYLINDERS*

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The hydrodynamic stability of the flow of liquid helium II between rotating cylinders has been examined theoretically by Chandrasekhar and Donnelly.[1] They showed that if one begins with a two-fluid set of equations of motion containing a linear coupling force (mutual friction) between the normal and superfluid components, there should be two critical velocities at which instability begins. The first of these to appear as the speed of rotation is increased is clearly associated with the motion of the superfluid component, the second with the normal fluid component of the liquid. The particular choice of the form of the coupling term determines the exact location of the two critical velocities.

In order to test this theory, a rotating cylinder viscometer has been built and has proved to be a very sensitive instrument for studying stability in ordinary liquids.[2,3] In particular, the onset of instability can be located within very small speed intervals and agrees with theory to within 1%.[2] The instrument, as used for the measurements reported here, is substantially the same as in reference 2 except that the null position is determined by means of a special interferometer which will be described in detail elsewhere.

This Letter reports preliminary measurements with helium II. In these experiments the deflection of the outer cylinder is denoted as ϕ and the period of rotation of the inner cylinder as P, so that in laminar flow the product ϕP is a constant proportional to the viscosity η of the liquid. The constant $C = \eta/\phi P$ is determined by calibration in air. In all cases reported here the cylinders have radii $R_1 = 1.9$ cm and $R_2 = 2.0$ cm. In Fig. 1 (a) is shown the result of measurements on liquid nitrogen which has been kept from boiling by a slight overpressure. The critical velocity is marked by an abrupt increase in ϕP at $P_c = 3.46$ sec. The slight rise in effective viscosity just before the critical period of rotation is observed in all ordinary liquids.[2] It amounts to less than 1% and cannot be seen in the helium measurements because of the small torques being measured.

The results at three different temperatures in liquid helium are shown in Figs. 1 (b), (c), and (d). It can be seen that the results are much different than in (a), particularly at the lowest temperatures. There appear to be three regions of interest. At long periods of rotation the viscosity is shear-independent. The value of viscosity corresponding to the horizontal line agrees satisfactorily with values obtained by Heikkila and Hollis Hallett[4] for the case where torque is measured on an inner cylinder instead of the present arrangement. The values of ϕP begin to rise at the speed denoted by P_s and tend to level off; the values of ϕP in this region are not very reproducible and may depend upon the rate of increase of speed of the motor. The amount of rise in effective viscosity varies with temperature,

being greatest at the lowest temperature. This suggests that the superfluid component is involved. At the speed indicated by P_n there is a sudden change of slope in the curve corresponding to that observed at P_c in (a). At P_n characteristic fluctuations of the mirror on the suspension are seen just as in ordinary liquids. The appearance of these curves is in many ways similar to the damping vs amplitude curves discussed by Donnelly and Hollis Hallett[5] for oscillating systems in helium II.

The preliminary results shown in Fig. 1 are consistent with the prediction of Chandrasekhar and Donnelly that there are two critical velocities in this experiment. The rise in ϕP at P_s is considered to be due to cellular motion associated with instability in the superfluid and the rise at P_n with cellular motion associated with instability in the normal fluid. The experiment is being continued at different temperatures and with different cylinder sizes. We hope also to increase the sensitivity of the viscometer so as to give greater accuracy at low speeds. It should then be possible to make quantitative comparisons with the theory.

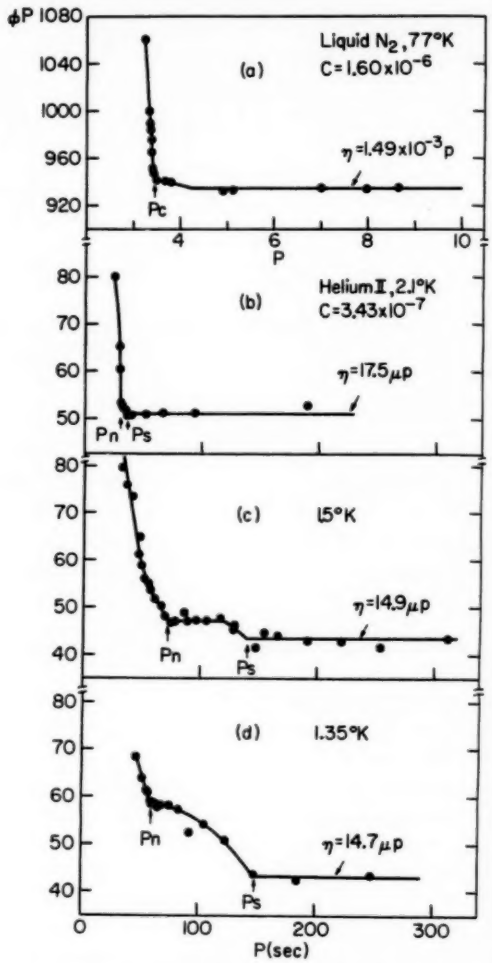

FIG. 1. The variation of ϕP, which is proportional to the effective viscosity, with period of rotation P of the inner cylinder. (a) Liquid nitrogen at 77°K; (b), (c), and (d) liquid helium II at three different temperatures. P_S and P_n denote the critical periods of rotation for instability due to the superfluid and normal fluid components.

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† Alfred P. Sloan Research Fellow.

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PARAMAGNETIC RESONANCE OF OXYGEN IN ALKALI HALIDES

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Alkali halide crystals grown from the melt exhibit a paramagnetic resonance spectrum at low temperatures which we have attributed to oxygen and have investigated in detail in the chlorides, bromides, and iodides of sodium, potassium, and rubidium. The concentration of these paramagnetic centers can be greatly enhanced by heating the crystals in an oxygen atmosphere, and it can be reduced to undetectable amounts by heating the crystals in other gases, such as hydrogen, halogen, water vapor, or in vacuum.

The paramagnetic resonance spectra indicate that the nuclei to which the resonant electron is bound do not give rise to hyperfine splitting and consequently must have spin 0, as does O^{16}. However, a weak hyperfine interaction with four equivalent alkali nuclei can be partially resolved under favorable circumstances in potassium and rubidium halides. The g tensor has orthorhombic symmetry with the axes parallel to [110], [1$\bar{1}$0], and [001]. These facts together rule out a monatomic center. The diatomic molecule ion O_2^- is the most probable model. It substitutes for a halide ion as shown in Fig. 1.

The ground state of O_2^- is a $^2\pi$ state according to the occupation of the levels given in Fig. 2.

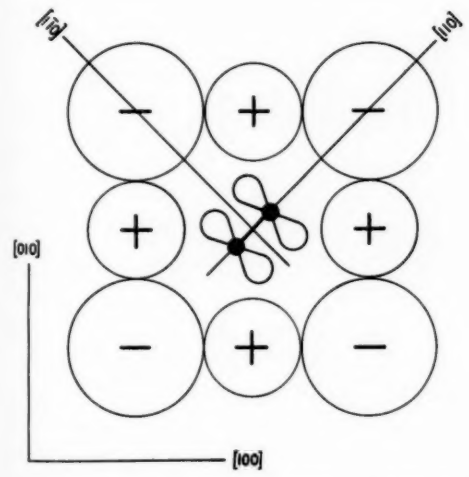

FIG. 1. An O_2^- molecule ion substituted for a halide ion in an alkali halide crystal. The orientation of the p function of the unpaired electron corresponds to the case of the potassium and rubidium halides.

The hyperfine interaction with four equivalent alkali nuclei indicates that in the potassium and rubidium halides the $p\pi$ orbital is oriented as shown in Fig. 1. This fact is substantiated by the analysis of the g tensor.

Explicit formulas for the g factors have been derived on the basis of the level scheme shown in Fig. 2. No assumption was made about the relative order of the spin-orbit splitting and the crystal field splitting. Mixing between the π_g levels and all others was included only in second order. Simplified versions of the formulas so obtained are

$$g_\zeta = g_e + 2\left(\frac{\lambda^2}{\lambda^2+\Delta^2}\right)^{1/2} l, \tag{1}$$

$$g_\xi = g_e\left(\frac{\Delta^2}{\lambda^2+\Delta^2}\right)^{1/2} - \frac{\lambda}{E}\left[-\left(\frac{\lambda^2}{\lambda^2+\Delta^2}\right)^{1/2} - \frac{\Delta}{(\lambda^2+\Delta^2)^{1/2}}+1\right], \tag{2}$$

$$g_\eta = g_e\left(\frac{\Delta^2}{\lambda^2+\Delta^2}\right)^{1/2} - \frac{\lambda}{E}\left[\left(\frac{\lambda^2}{\lambda^2+\Delta^2}\right)^{1/2} - \frac{\Delta}{(\lambda^2+\Delta^2)^{1/2}} - 1\right], \tag{3}$$

where ξ is chosen along the $p\pi$ function, and ζ is the internuclear axis. Here λ is an effective spin-orbit splitting of the molecule ion in the

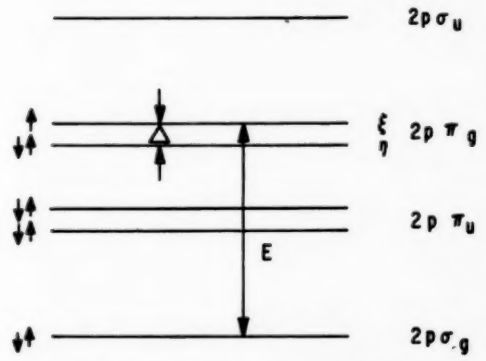

FIG. 2. Occupation of the electronic levels of O_2^- in the ground state.

crystal, Δ is the $2p\pi_{g\xi}$-$2p\pi_{g\eta}$ separation and E is the $2p\pi_g$-$2p\sigma_g$ separation. Further, l represents a correction to the angular momentum about ζ caused by the crystal field; l is unity for the free molecule ion. The first two terms in (1) and the first term in (2) and (3) are exact. All second-order terms except those from the $2p\sigma_g$ level have been omitted. The matrix elements in the second-order terms kept have been replaced by those appropriate to the isolated molecule ion.

The calculation shows that g_η exceeds g_ξ. From the experimental g tensors we can thus infer that the $p\pi$ functions point along $[1\bar{1}0]$ in the potassium and rubidium halides (as depicted in Fig. 1), but along [001] in the sodium salts. These assignments are consistent with the hyperfine data.

A typical example is furnished by KCl with $g_\zeta = 2.4359$, $g_\xi = 1.9512$, and $g_\eta = 1.9551$. The corresponding values of λ/Δ and λ/E are 0.23 and 0.0025 and l is 1.04. A value of λ taken from atomic oxygen, 226 cm^{-1}, when combined with these values for λ/Δ and λ/E, leads to separations Δ and E about four times larger than might be expected in comparison with the well-investigated $(halogen)_2^-$ molecule ions.[1,2] However, in O_2^- the predominant contribution to λ arises probably from an O^- configuration which would result in a λ several times smaller than for atomic oxygen. This would lead to quite reasonable values of Δ and E.

The optical absorption near 4.9 ev observed in similarly treated crystals by Rolfe,[3] and the associated fluorescence emitted between 1.8 ev and 2.8 ev studied by Honrath[4] and Ewles and Barmby[5] may well be due to this O_2^- center. However, confirmation of this proposal would require simultaneous study of optical absorption and paramagnetic resonance.

We are grateful to J. Owen and T. G. Castner, Jr., for their important contributions to the theory of the g factor, to J. Rolfe for communication of the optical absorption spectrum, and to F. E. Williams for discussion of the absorption and fluorescence spectrum.

A full account of the present work will be published in a forthcoming paper.

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DEPENDENCE OF SECONDARY ELECTRON EMISSION ON CRYSTAL ORIENTATION

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The dependence of secondary electron emission on the angle of incidence of primary electrons has been examined using single crystals of MgO and LiF. An unexpected dependence on crystal orientation has been observed. A series of maxima and minima occurs superposed on the previously reported[1] increase in secondary emission with increasing angle of incidence. The maxima have been found to represent as much as 20% of the total secondary emission yield under some conditions and therefore contribute substantially to the total number of secondary electrons. The origin of the maxima and minima is not well understood. However, they do not appear to result from a diffraction effect since the angular positions of the maxima are not dependent on the energy of the primary electrons.

The secondary emission measurements were made using sealed-off tubes under clean high-vacuum conditions with pressures of 10^{-9} mm Hg or less. The tubes included an electron gun producing a collimated electron beam of 3° divergence, a graphite-coated secondary electron collector, and a stabilizing filament for flooding the crystals with low-energy electrons. A target holder was suspended from two rods through the side of the tube so that by rotating the tube about the axis of the rods the angle of incidence could be varied. The crystal and target holder remained fixed in position under the influence of gravity.

The MgO and LiF single crystals were cleaved

along a (100) plane immediately prior to insertion in the vacuum tube as described elsewhere.[2] To minimize charging of the insulating crystal surfaces single pulse measurement techniques previously described[3] were employed. The incident charge per pulse could be limited to 2×10^{-14} Coulomb and the change in the target surface potential to about 0.1 volt. Between measurements the target surface potential was reproducibly established by flooding the crystal with low-energy electrons from the stabilizing filament.

Figure 1 shows typical curves of secondary electron yield of MgO and LiF as a function of the angle of incidence of the primary electrons. For these data the crystals were oriented with the normal to the surface being the [100] direction. Rotation was about the [010] direction. The curves for both MgO and LiF show pronounced maxima around 0° and 45° angles of incidence. The angular positions of these maxima are independent of primary electron energy. For both MgO and LiF several other small maxima are apparent. The exact angular positions of the maxima are difficult to determine since they are superposed on the generally observed smooth increase of yield with angle of incidence.[4] The curves are symmetrical about 0° (normal incidence) and only one direction of rotation is presented here.

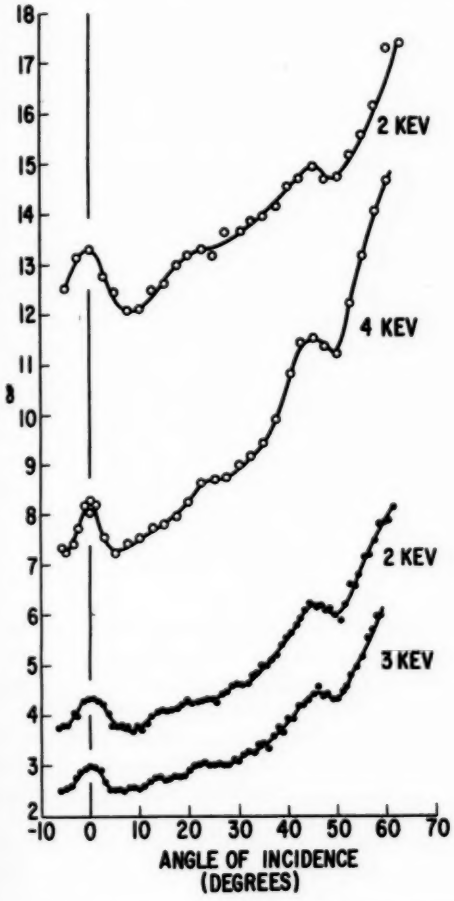

FIG. 1. Dependence of total yield on angle of incidence of the primary electrons with the indicated energies. Axes of rotation are [010] directions while the crystal surfaces are (100) planes. Curves are shown for one direction of rotation. Symmetry exists about 0° angle of incidence. Open circles correspond to data obtained on MgO and solid circles to data obtained on LiF.

A second type of measurement also indicates the importance of crystal orientation on the secondary electron emission process. The primary electron beam was held at a fixed angle of incidence with respect to the surface of the crystal while the crystal was rotated about the [100] direction normal to the surface. Figure 2 shows the yield of MgO as a function of the angle of rotation of the crystal about the [100] direction normal to the surface. The primary beam was fixed at 45° angle of incidence. Maxima occur at 0° and 90° which correspond to ⟨110⟩ directions. A broad maximum occurs at about 45°. The angular positions of the maxima again appear to be independent of the primary electron energy.

In both experiments the angular positions of the large maxima correspond to directions where

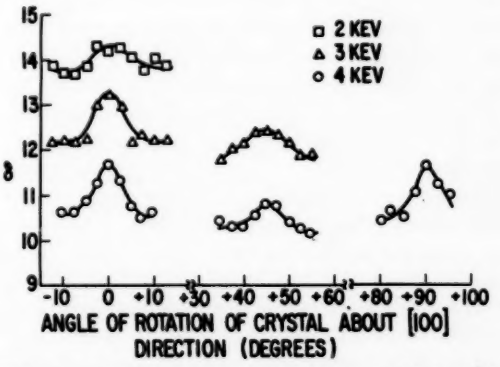

FIG. 2. Dependence of the yield from an MgO single crystal as a function of the angle of rotation of the crystal about the [100] direction normal to the surface. The primary beam was held at a constant 45° angle of incidence with respect to the normal to the surface.

the primary beam is incident along low-index crystalline directions (close atomic spacing) and where a slight angular deviation corresponds to a high-index direction. In Fig. 1 the two large maxima at 0° and 45° correspond to the [100] and [110] directions, respectively. In Fig. 2 the maxima at 0° and 90° correspond to the [110] and [011] directions. There are several relatively low-index directions in the vicinity of 45° lying within the 3° divergence of the primary beam. These probably account for the broad maximum observed in this region.

The authors wish to express their thanks to V. L. Stout, J. R. Young, and Professor W. G. Shepherd for many stimulating discussions of this work.

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TEMPERATURE DEPENDENCE OF THE ABSORPTIVITY OF COPPER IN THE NEAR INFRARED

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The temperature dependence of the absorptivity of the noble metals in the near infrared can yield valuable information regarding electron-phonon interactions in these metals, as well as a crucial test of the effect of electronic surface collisions on their optical properties. To date there have been no reliable measurements of this kind. The present communication is a preliminary report of absorptivity measurements covering the temperature range 77-300°K, on carefully annealed and electropolished bulk copper.

A calorimetric technique, shown schematically in Fig. 1, is employed in this work. Infrared radiation is obtained from a 1000-watt projection lamp in conjunction with a long-wavelength-pass interference filter having a cutoff at 1.8 microns. The maximum wavelength of the pass band is limited by the glass envelope of the projection lamp to about 4 microns. A spherical mirror focuses the radiation onto the copper target, which is in the form of a thin disk of electropolished metal about $\frac{1}{2}$ inch in diameter and 15 mils thick. The reflected energy is collected on an absorber coated with palladium black, which has substantially zero reflectance in this spectral region. Both the target and absorber are supported by very thin copper tubes so that, with radiation incident on the system, temperature differences are set up across the supports. These temperature differences are detected by copper-constantan differential thermocouples T, which actuate Liston-Becker breaker amplifiers. By reproducing the temperature rises with dc power from the heaters H, the absorptivity of the copper target may then be obtained from the relation $A = P_t/(P_a + P_t)$, where P_t and P_a are the dc power inputs to the target and absorber, respectively. A copper-constantan thermocouple attached to the bottom of the inner Dewar measures the average temperature of the specimen. This is varied by judicious cooling of the inner Dewar, followed by subsequent evacuation.

The results of the experiments are shown in Fig. 2. At 300°K, the value for the absorptivity of copper is 0.76%. This figure is in good agreement with that obtained by Schulz[1] using evaporated films, but differs markedly from previous data on bulk material.[2] The difference is presumably due to improved experimental technique, particularly as regards surface preparation. At low temperatures, the absorptivity is substantially constant at a value of 0.44%, in good accord with the value obtained at liquid helium temperatures.[3]

In general the absorptivity is given by the equation

$$A = A_v + A_s, \tag{1}$$

where A_v and A_s arise from scattering of the electrons by phonons and the surface, respectively. For the near infrared ($\omega\tau \gg 1$) and for $T \gg \theta$, the volume absorptivity A_v has the classical value

$$A_v^{\mathrm{cl}} = \langle\tau\rangle\langle 1/\tau\rangle(Ne^2/\pi m)^{1/2}\rho, \tag{2}$$

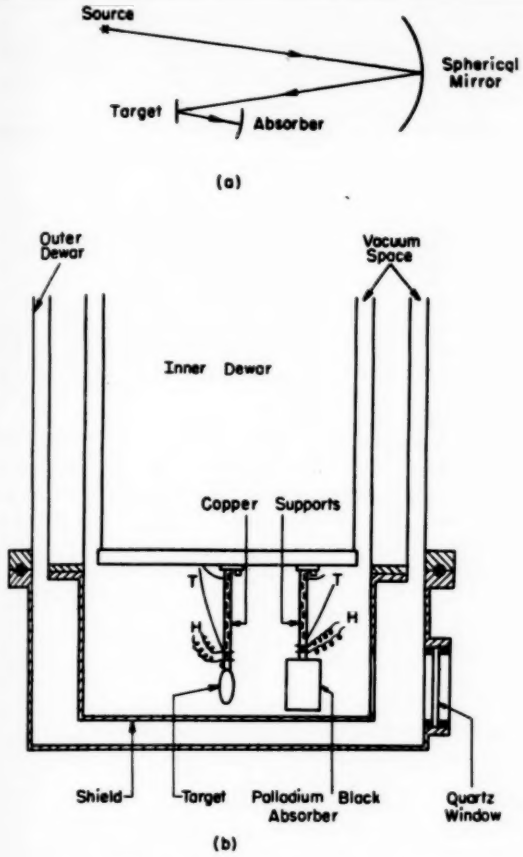

FIG. 1. Optical absorption apparatus. (a) Optical path; (b) details of cryostat.

where $\langle\tau\rangle$ and $\langle 1/\tau\rangle$ are integrals over the Fermi surface of the phonon-electron relaxation time τ, viz.,

$$\langle\tau\rangle = \int \tau v dS \Big/ \int v dS; \quad \langle 1/\tau\rangle = \int (1/\tau) v dS \Big/ \int v dS. \quad (3)$$

The Schwartz inequality gives $\langle\tau\rangle\langle 1/\tau\rangle \geq 1$, the equality holding only for isotropic scattering.[3]

When $\hbar\omega \gg k\theta$, quantum effects[4] modify the volume absorptivity, which then has the form

$$A_v = A_v^{\mathrm{cl}} \frac{\theta}{T}\left\{\frac{2}{5} + \frac{4/5}{e^{\theta/T}-1} + \frac{4}{5}\left(\frac{T}{\theta}\right)^5 \mathcal{J}_5\left(\frac{\theta}{T}\right)\right\}, \quad (4)$$

where

$$\mathcal{J}_5(x) \equiv \int_0^x \frac{z^5 dz}{(e^z-1)(1-e^{-z})}.$$

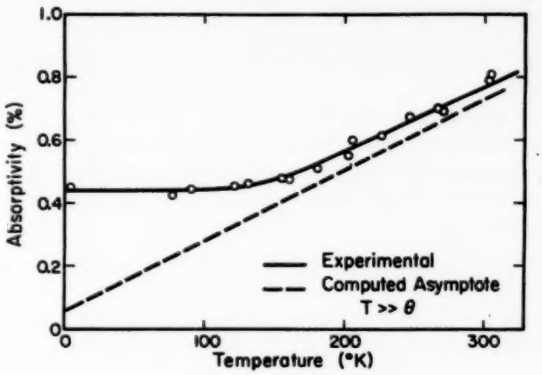

FIG. 2. Variation with temperature of infrared absorptivity of copper.

This expression reduces to (2) for $T \gg \theta$, but at low temperatures gives

$$A_v = \tfrac{2}{5} A_v^{\mathrm{cl}} \theta/T. \quad (5)$$

At intermediate temperatures, an expansion of (4) yields

$$A_v = A_v^{\mathrm{cl}}\{1 + \tfrac{1}{18}(\theta/T)^2\}. \quad (6)$$

This equation has been used to find the asymptotic dependence of the absorptivity. For $\theta = 315°\mathrm{K}$ the dashed line of Fig. 2 is obtained. Using the known values of the resistivity ρ and the effective optical mass m, Eq. (2) then gives

$$\langle\tau\rangle\langle 1/\tau\rangle = 1.64, \quad (7)$$

i.e., the phonon-electron relaxation time is not isotropic as is usually supposed.

From the intercept of the asymptote on the vertical axis, one obtains at $T = 0$

$$A_v = 0.0038, \quad A_s = 0.0006. \quad (8)$$

Thus, experimentally for $T = 0$

$$A_v = 0.56\, A_v^{\mathrm{cl}} \theta/T. \quad (9)$$

The discrepancy between this value and that given by (5) in part comes from the assumption of acoustic isotropy and spherical energy surfaces in deriving (4). Neither of these conditions is fulfilled in the case of copper. Another cause of the difference is that we have assumed θ to be the calorimetric value. Actually we should use the longitudinal Debye temperature ($\sim 1.5\theta_{\mathrm{cal}}$).

If we take $\theta = \theta_l = 450°K$,

$$A_v = 0.4\, A_v^{\mathrm{cl}} \theta_l / T, \tag{10}$$

in agreement with theory.

The value of the surface contribution is much smaller than has previously been supposed.[5] It has been shown[6] that A_S is given by the expression

$$A_S = (1-p) \times \tfrac{3}{4}(v/c), \tag{11}$$

where p is the fraction of electrons specularly reflected from the surface, and v is an average of the Fermi velocity of the electrons. If it is assumed that there is diffuse scattering of the electrons ($p = 0$), Eq. (11) gives $A_S = 0.0027$ when one takes $v = 1.55 \times 10^8$ cm sec^{-1}. The present data show that this is far too large and hence that we must suppose $p \sim 1$ to obtain agreement with theory, i.e., the scattering of the electrons by the surface at optical frequencies is predominantly specular. This behavior is to be contrasted with that at microwave frequencies, where the scattering has been shown[7] to be essentially diffuse. No explanation of this discrepancy can be advanced.

Measurements on silver and gold are currently being made and will be reported in the near future.

The author gratefully acknowledges many helpful discussions with Dr. T. Holstein regarding this work.

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NEW METHOD FOR THE EVALUATION OF h/e FROM THE QUANTUM LIMIT OF THE CONTINUOUS X-RAY SPECTRUM

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Planck's constant, h, is one of the least accurately determined of the physical constants. This fact is all the more surprising, as there exists a quite accurate method among the so-called "h/e experiments" contributing most to its determination. This method is the determination of the short-wavelength limit (SWL) of the continuous x-ray spectrum.[1-3] Unfortunately, this method suffers from a serious systematic error of hitherto unknown source.[4,5] Cohen and DuMond[4] and Bearden and Thomsen[5] suspected the origin of the discrepancy between this and all other determinations of h to lie in the ignorance of the exact shape of the SWL, and in the associated inability for identifying the true position of the threshold.

It will be shown, in this method, how these twenty year old discrepancies may be resolved by means of the isochromat measurements which we have made. We will also show that the SWL experiments agree with the majority of the other experiments. This problem is also important for the new determinations of some of the input data for planned readjustments of the atomic constants.[6]

The investigations of the SWL are describable in terms of the measurements of isochromats, i.e., the voltage applied to the x-ray tube is varied in small steps, and the intensity for a fixed wavelength setting of a monochromator is measured. For the small wavelength range of interest, this physical situation leads to a scale replica of the spectral intensity curve in the vicinity of the SWL. The problem is then to find out the exact position of the threshold voltage belonging to the wavelength setting of the monochromator. Up to now, the usual procedure has been to identify this position with that of the "point of maximum bending" (maximum of the second derivative) of the observed isochromat.[4,5,7]

The implicit model, in Sommerfeld's theory of bremsstrahlung, which has previously applied, is that of an encounter of an electron with a stripped, isolated nucleus in the so-called free-

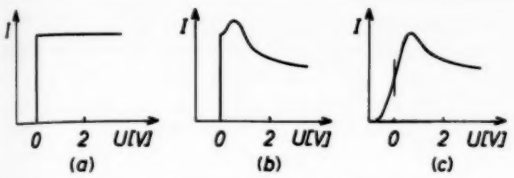

FIG. 1. Ideal W isochromats. (The abscissa is the difference between tube voltage and threshold voltage; nominal voltage ≈ 1250 v.) (a) According to Sommerfeld's theory of bremsstrahlung; (b) allowance made for the band structure of tungsten ($T = 0°K$); (c) allowance made for the finite temperature of the anticathode.

free transition. The resulting form of the ideal isochromat for monochromatic electrons and a thin anticathode is shown in Fig. 1(a), where the isochromat is considered measured with infinitely good resolution. The physical situation described by Sommerfeld's theory is not quite accurate, because any real anticathode is made of a metallic sheet. One has to take into account the special properties of the solid state. The electrons having generated an energetic quantum in the neighborhood of the SWL must be located in the nonoccupied part of the conductivity band. The distribution of these free positions determines the energy distribution of the continuous spectrum at the SWL and therefore also the shape of the isochromat. Now we make the first assumption that the transition probabilities to the final states of the electrons are equal for all free positions in the nonoccupied regions of the conductivity band. Then the distribution of these positions gives directly the shape of the ideal isochromat. This is carried out in Fig. 1(b) for the band structure of tungsten by Manning and Chodorow.[8] The Fermi edge of W corresponds to the threshold voltage. It should be emphasized here that our final result depends only to a very small degree on the details of the band structure. In Fig. 1(c) allowance is made for the high temperature of the anticathode during our measurements resulting in a broadening of the Fermi edge. We make now the second essential assumption that we obtain the thin anticathode spectrum within about the first ten volts from threshold even for the really thick anticathode used for the measurements. This assumption, though surprising at first sight, is supported quite well by the distribution of the characteristic energy losses of the electrons in W.[9,10] With this conception the measured isochromat differs from

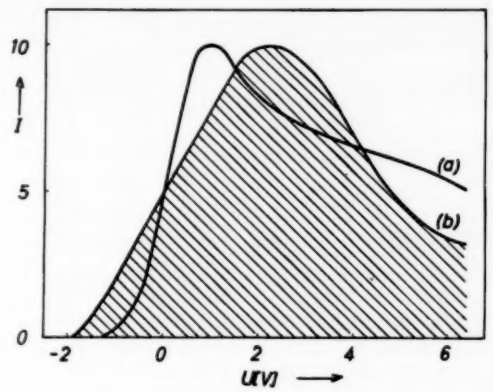

FIG. 2. Theoretical (a) and experimental (b) isochromat, normalized to same height of maximum. (Abscissa as in Fig. 1.)

that of Fig. 1(c) only due to the finite resolving power of the experimental apparatus. We obtain the "instrumental window" of our experiment by folding up numerically the following influences: (1) the monochromator pass-band; (2) the fluctuations of the x-ray tube voltage; (3) the energy distribution of the emitted electrons. Now this instrumental window is folded with the isochromat of Fig. 1(c). Having considered all influences correctly we should obtain a curve in conformity with our measured isochromat. Fig. 2(a) shows the calculated isochromat and (b) the measured one. The resemblance is quite satisfactory. The remaining deviation of the rise to the maximum has its origin probably in the limited accuracy of the calculation of the resolving power.

For our problem the most important information in Fig. 2 is that concerning the threshold voltage position. It is simply given by the abscissa belonging to the ordinate with half the value of the maximum. The consequences of this result are contained in Table I (corresponding to Table VII of reference 4). It shows the conparable SWL experiments since 1950. Comparable in this sense means measurements with W anticathodes and W equipotential cathodes. The discrepancy of the last two columns is the deviation of the threshold voltage in the SWL experiments from the threshold voltage required by the bulk of the other experiments, first for the previous evaluation method and then for the new conception. The sum of discrepancies in the last column disappears below the error limits

Table I. X-ray quantum limit determinations.

Experiment	Nominal voltage (volts)	Resolution[a] (volts)	Discrepancy (volts) old	Discrepancy (volts) new
BS[b]	8050	0.75	-1.4	+1.5
BS[b]	19600	1.7	-3.4	-1.4
BJW[c]	6112	0.55	-0.7	+0.4
BJW[c]	10168	0.95	-0.8	+0.4
		Sum of discrepancies	-6.3	+0.9
FHD[d]	24500	12	-4.3	+3.4

[a]Full half-width.
[b]J. A. Bearden and G. Schwarz, Phys. Rev. 79, 674 (1950).
[c]Bearden, Johnson, and Watts, Phys. Rev. 81, 70 (1951).
[d]Felt, Harris, and DuMond, Phys. Rev. 92, 1160 (1953).

of the measurements. Due to the bad resolution, the newest investigation in the lowest line does not fulfill our second assumption and therefore is not taken into account. It should be mentioned that values of the last column contain also a small correction for the mean velocity of the emitted electrons as first recognized by Bearden and Thomsen.[5]

I am grateful to H. Vernickel for the measurements, and to Dr. G. Laukien for making possible the publication of this paper. A full account of this work will be published elsewhere.

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THEORY OF THE VECTOR INTERACTION WITH A CONSERVED CURRENT AND THE BETA DECAY OF Na^{24}-Al^{24} NUCLEI

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In the theory of the vector interaction with a conserved current the observed deviations from the $\Delta T = 0$ selection rule in Fermi transitions have to be explained only in terms of isotopic spin impurities.[1] However, in the conventional theory exchange mesonic currents may also induce Fermi transitions with $\Delta T \neq 0$. Recently an attempt[2] has been made to estimate the contribution of isotopic spin impurities introduced by the Coulomb interaction between the protons. The relevant Coulomb matrix elements have been calculated with the wave functions given by the j-j coupling shell model. A comparison between the calculated Fermi matrix element and the experimental one was performed in the case of the β^- decay of the $T=1$, $J=4^+$ state of Na^{24} to the state $T=0$, $J=4^+$ of Mg^{24}. The experiments[3] on Na^{24} agree only with a value of M_F smaller than 10^{-3} while the theoretical estimate yields a value of about 1.3×10^{-2}. The two following interpretations are possible: (a) By using j-j coupling shell model wave functions the Coulomb matrix ele-

ment has been overestimated; (b) exchange mesonic currents do induce Fermi transitions, and a cancellation occurs between the Coulomb term and the mesonic term. In this note we discuss a possible experiment which would enable us to make a choice between these two possibilities without having to use any nuclear model.

Let us consider the $T=1$ multiplet which consists of the ground states of Al^{24} and Na^{24} and of the 9.5-Mev level of Mg^{24}. The β^+ decay of the ground state of Na^{24} has a 99.9% branch to the $T=0$, $J=4^+$ level of Mg^{24} and the β^- decay of Al^{24} has a 10% branch to the same level. The Fermi matrix element for these two transitions is given by

$$M_F^{\pm}=\langle\alpha_f\ T=T_\zeta=0|\mathcal{K}_F^{\mp}|\alpha_i\ T=1\ T_\zeta=\pm1\rangle + a_0(1)\langle\alpha_i\ T=1\ T_\zeta=0|\mathcal{K}_F^{\mp}|\alpha_i\ T=1\ T_\zeta=\pm1\rangle. \quad (1)$$

In this formula $\pm$ refers to $\beta^{\mp}$ decay. The operator $\mathcal{K}_F^{\mp}$ which describes the Fermi transition is given in the conventional theory by

$$\mathcal{K}_F^{\pm}=(C_V^0/C_V)\mathcal{T}^{\pm}=(C_V^0/C_V)\int d^3x\ \bar{\psi}\tau_{\pm}\psi, \quad (2)$$

where C_V^0 is the bare coupling constant and C_V the renormalized one.

In the theory with a conserved current, we have

$$\mathcal{K}_F^{\pm}=T^{\pm}=\mathcal{T}^{\pm}+\mathcal{R}^{\pm}=\mathcal{T}^{\pm}+i\int(\phi^*\rho_{\pm}\frac{\partial\phi}{\partial t}-\frac{\partial\phi^*}{\partial t}\rho_{\pm}\phi)d^3x. \quad (3)$$

The operators $T^{\mp}$, $\mathcal{T}^{\mp}$, $\mathcal{R}^{\mp}$ are, respectively, the total isotopic spin operator, the nucleon isotopic operator, and the π-meson isotopic spin operator. The indices α_f, α_i represent the set of quantum numbers characterizing the initial and final states in the absence of a charge-dependent perturbation. The quantity $a_0(1)$ is the amplitude of the state $|\alpha_i\ T=1\ T_\zeta=0\rangle$ which is mixed to the state $|\alpha_f\ T=0\ T_\zeta=0\rangle$ under the influence of the charge-dependent perturbation. Now we apply the Eckart-Wigner theorem to the isotopic vector operator $\mathcal{K}_F^{\mp}$. We obtain

$$M_F^{\pm}=\pm\langle T=0\|\mathcal{K}_F\|T=1\rangle+a_0(1)\langle T=1\|\mathcal{K}_F\|T=1\rangle. \quad (4)$$

Inserting into the above formula the expression for $\mathcal{K}_F$, we get in the conserved current theory

$$M_F^{\pm}=a_0(1)\sqrt{2}, \quad (5)$$

and in the conventional theory

$$M_F^{\pm}=\mp(C_V^0/C_V)\langle T=0\|\mathcal{R}\|T=1\rangle+a_0(1)\sqrt{2}. \quad (6)$$

In the formula (6) we have replaced $(C_V^0/C_V)\times\langle T=1|\mathcal{T}^{\pm}|T=1\rangle$ by $\sqrt{2}$, which means that we neglect possible exchange effects in the $\Delta T=0$ matrix element. This is quite legitimate since we know from the ft values of $0^+\text{–}0^+$ transitions that the Fermi matrix element does not differ from $\sqrt{2}$ by more than a few percent.

Let us now discuss the angular distribution of the polarized γ ray of Mg^{24*} relative to the direction of emission of the $\beta^{\mp}$ particle:

$$W^{\pm}(\theta)d(\cos\theta)=[1+(v/c)\tau A^{\pm}\cos\theta]d(\cos\theta), \quad (7)$$

where $\tau=\pm1$ stands for right (left) circular polarization. The anisotropy coefficient A is given by

$$A^{\pm}=\tfrac{1}{3}[\pm\tfrac{1}{4}+(\sqrt{5})y^{\pm}]\frac{1}{1+(y^{\pm})^2}, \quad (8)$$

with

$$y^{\pm}=|(C_V/C_A)|(M_F^{\pm}/M_{GT}^{\pm}).$$

A formula similar to (4) is valid for $M_{GT}^{\pm}$:

$$M_{GT}^{\pm}=\pm\langle\alpha_f\ T=0\|\vec{\mathcal{K}}_{GT}\|\alpha_i\ T=1\rangle + a_0(1)\langle\alpha_i\ T=1\|\vec{\mathcal{K}}_{GT}\|\alpha_i\ T=1\rangle, \quad (9)$$

where

$$\vec{\mathcal{K}}_{GT}=\int d^3x\ \bar{\psi}\beta\vec{\sigma}\psi.$$

The matrix element $\langle\alpha_i\ T=1\|\vec{\mathcal{K}}_{GT}\|\alpha_i\ T=1\rangle$ has been estimated by Bolsterli and Feenberg[4] to be about 0.15. From the ft value of Na^{24} we know that $\langle\alpha_f\ T=0\|\vec{\mathcal{K}}_{GT}\|\alpha_i\ T=1\rangle$ is of the order of 0.06. So even if $a_0(1)$ is taken equal to the calculated value 0.009, we may neglect the second term in the formula (9) and use for

$$\langle\alpha_f\ T=0|\vec{\mathcal{K}}_{GT}|\alpha_i\ T=1\rangle$$

the value deduced from the ft value of Na^{24}. Using the formula (5) we see that if the theory of the vector interaction with a conserved current is the correct one the following equation holds:

$$A^{(+)}+A^{(-)}=0. \quad (10)$$

As a test of the theory of the vector interaction

with a conserved current we suggest the repetition on Al^{24} of the experiment already done on Na^{24} by several groups. If the relation (10) happens to be true we predict for Al^{24} an anisotropy coefficient $A^{(-)}$ close to the pure Gamow-Teller value, namely, $A^{(-)} = -0.08$. Such a result would also imply that the theoretical estimate of $a_0(1)$ was incorrect. But if the interpretation (b) is the right one a very different value for $A^{(-)}$ will be found. In that case we assume that the calculated value for $a_0(1)$ is correct and that a mesonic term exists which cancels amost exactly the Coulomb term [Eq. (6)]. We will find for the anisotropy coefficient $A^{(-)}$ the two following values depending on the sign of the ratio $a_0(1)/M_{GT}^{(+)}$ (a j-j coupling calculation suggests a positive sign):

$$A^{(-)} = -0.32, \quad [a_0(1)/M_{GT}^{(+)}] > 0,$$

$$A^{(-)} = 0.17, \quad [a_0(1)/M_{GT}^{(+)}] < 0. \qquad (11)$$

We would like to add a final remark concerning the validity of the formulas (7) and (8). Since the $\beta^{(+)}$ decay branch of Al^{24} we are considering is a high-energy branch ($E_{max} \simeq 9.5$ Mev) with a large ft value ($\log ft \simeq 6.1$), the forbidden correction may not be completely negligible. However, we have made an estimate of these corrections and we have found that they are negligible compared to the experimental errors in the measurment of the circular polarization of the γ ray.

We are indebted to Professor Treiman for a valuable criticism. We wish to express our appreciation to Professor Wigner for his encouragment and for reading the manuscript. We want also to acknowledge helpful discussions with Dr. Henry Hill and Professor Sherr about the feasibility of the experiment.

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SEARCH FOR Li^4†

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The possibility that Li^4 might be stable against decay into He^3 and a proton has led to revived speculation[1] concerning the effect which such a nucleus would have in stellar processes. Although there are good theoretical and some experimental arguments[1,2] against the existence of a β-active Li^4, it seemed important to make a direct, experimental investigation of this nucleus. Li^4, if just particle-stable, would be converted into He^4 by emitting a positron with an end-point energy near 19 Mev. The mean life of Li^4 may be estimated from calculations[3] on the decay of the mirror nucleus, H^4, to be in the neighborhood of 30 milliseconds. Consequently, it was decided to try to produce Li^4 in the reaction $He^3(p,\gamma)Li^4$, and to detect the residual nucleus by counting the delayed positrons from $Li^4(\beta^+\nu)He^4$.

Figure 1 illustrates the target arrangement.

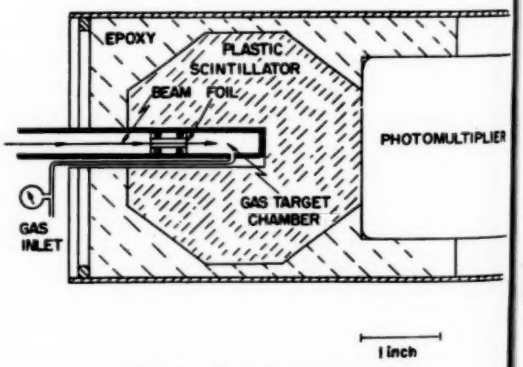

FIG. 1. Target arrangement.

Either He^3 (90% pure) or He^4 (assumed 100% pure) served as the target gas at an absolute pressure of 25 psi. Protons were accelerated

an energy of 1.37 Mev. The entrance foil was 160 kev thick and the gas target 140 kev thick, so that the protons interacted with the He^3 with a mean energy of 1.14 Mev or $E_{c.m.} = 855$ kev. The responses of the heavily shielded scintillator to cosmic rays, 6-Mev gamma rays from $F^{19}(p,\alpha\gamma)O^{16}$, pairs from $F^{19}(p,\alpha\pi)O^{16}$, 2.62-Mev gamma rays from ThC'' and betas from $B^{11}(d,p)B^{12}(\beta\nu)C^{12}$ were determined, and the detector and scalers biased to give an over-all detection efficiency of 80% for the positrons expected from Li^4.

An electromagnet, operated at 60 cps, swept the proton beam past a narrow slit, allowing a beam pulse as shown in Fig. 2 to strike the target. Two scalers, together comprising a single-channel analyzer spanning the appropriate pulse heights, were gated to count while the beam was off, as illustrated in Fig. 2. It was found that the number of counts was independent of the target gas, and came from machine background and cosmic rays. With the chopping cycle of Fig. 2, a total of 2745 counts was recorded in 7×10^5 cycles with He^3 in the target chamber. The total charge collected was 6800 microcoulombs. After subtraction of the background counts, which were identical within statistics to the above number, an upper limit may be placed on the cross section for formation of Li^4. That limit depends on the assumed mean life. For data taken in the described manner, one may write

$$N = N_0 N_1 \epsilon \sigma F(\lambda) B(t_1),$$

where N = total number of counts (N_{He^3}) corrected for background (N_{He^4}),

$$N = N_{He^3} - N_{He^4} \pm (N_{He^3} + N_{He^4})^{1/2} \leq (N_{He^3} + N_{He^4})^{1/2}.$$

N_0 = number of target nuclei per cm^2; N_1 = number of incident protons; ϵ = over-all detection efficiency; σ = cross section; λ = disintegration constant;

$$F(\lambda) = \frac{e^{\lambda t_1}(e^{-\lambda t_2} - e^{-\lambda t_3})}{\lambda t_1} \frac{(1 - e^{-\lambda t_1})}{(1 - e^{-\lambda t_4})} \left[1 - \frac{1 - e^{-m\lambda t_4}}{m(e^{\lambda t_4} - 1)}\right],$$

where m = number of cycles; t_1, t_2, t_3, t_4 are defined in Fig. 2, and

$$B(t_1) = \text{beam factor} = [1 + (\lambda t_1/2\pi)^2]^{-1}.$$

With these expressions and the measurements on N, N_0, N_1, ϵ, and $B(t_1)$, it is possible to calculate an upper limit for σ as a function of $\tau = 1/\lambda$. The results are not very sensitive to the beam shape, and the assumed beam factor

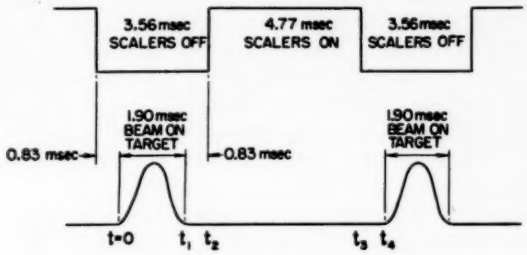

FIG. 2. Chopping cycle.

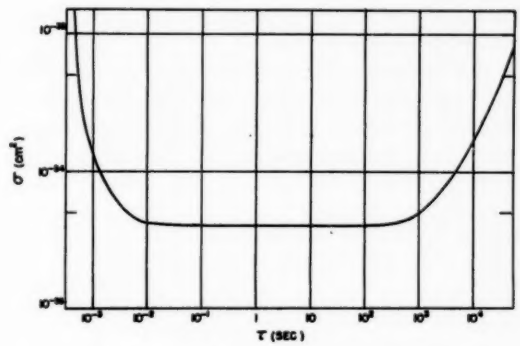

FIG. 3. Upper-limit cross section vs mean life.

is based on a sinusoidal approximation to the beam pulse. The results are shown in Fig. 3; for a mean life in the range 0.006 to 600 seconds, the cross section is $\lesssim 4\times10^{-11}$ barn.

Christy[4] has made a rough calculation of the theoretical cross section for $He^3(p,\gamma)Li^4$. That theoretical value, which is based on the reasonable assumptions of s-wave proton capture and a Li^4 ground-state assignment of 1^- or 2^-, depends on the assumed binding energy. If Li^4 were barely bound, σ should be in the neighborhood of 10^{-7} barn. A binding energy of 1 Mev would introduce some cancellation in the matrix element, and give the smaller value of 10^{-8} barn. Christy estimates that, if Li^4 were bound, one could expect $\sigma > 10^{-9}$ barn, which is greater by 25 times than the experimental limit, assuming a reasonable mean life for Li^4. Thus, one can assert that Li^4 is not bound.

This conclusion, which is based on a theoretical argument equally with the experimental finding, is extremely important in connection with the application to astrophysics. Were Li^4 actually bound, and were the upper limit of 4×10^{-11} barn the actual $He^3(p,\gamma)Li^4$ cross section, the cross

section factor[5] would be 2.3×10^{-7} kev-barn. In such an instance, the stars would consume He^3 via the $He^3(p,\gamma)Li^4$ process rather than by the $He^3(He^3,He^4)2p$ reaction. Indeed, a cross section 10^{10} times smaller than the present experimental upper limit would enable the $He^3(p,\gamma)Li^4$ reaction to compete with the direct process $He^3(p,\beta^+\nu)He^4$, which[6] is of significance at very low temperatures ($T<2\times10^6$ °K). Thus, the observations reported in this paper cannot absolutely deny an astrophysical role to Li^4, but the implication of the data and theory is strong that Li^4 is not particle-stable and hence does not in fact have such a role.

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MEASUREMENT OF THE SPIN AND PARITY OF THE ANOMALOUS INELASTIC STATES IN Ni^{58} AND Ni^{60}†

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In the inelastic scattering of charged particles of intermediate energy as measured with low-resolution detectors, a number of experimenters have observed structure in the spectra at an excitation energy where one previously would have expected only a continuum.[1-3] The details of the structure observed by the different experimenters depend rather critically on the precise resolution used, but generally speaking these experiments have shown a strong peak corresponding to leaving the nucleus excited to about 2.5 Mev for elements with $Z \geq 30$. Further structure is seen particularly for elements with $Z < 30$ at an excitation energy of about 4.5 Mev. In this note we wish to report on the experiments establishing the spin and parity of the 4.5-Mev structure in Ni^{58} and Ni^{60} as 3^-.

Figure 1 shows the angular distributions of alpha particles scattered from Ni^{58} and Ni^{60}, respectively, leaving the target nucleus in the ground state and first excited state. Also shown in these figures are the angular distributions of the alpha particles which leave the nucleus with the excitation indicated and constitute the anomalous peak at the respective energies. The spectra from which these angular distributions were deduced are similar to those published earlier by Sweetman and Wall.[3] The scintillation spectrometer used for experiments was a NaI crystal cut sufficiently thin (~0.02 inch) so that the largest proton or deuteron light pulse produced in the crystal was smaller than the alpha-particle pulses of interest.

In a subsequent experiment a 3×3 inch NaI γ-ray spectrometer was used to measure the γ-ray spectrum in coincidence with the anomalous alpha-particle groups. A fast-slow coincidence circuit was used that enabled us to simultaneously measure the elastic alpha-particle–γ-ray counting rate as well as the inelastic–γ-ray coincidence rate. This enabled us to show that the chance coincidence rate was in fact practically negligible and to make any minor correction (< 10%) for it. Details of this scheme will be published later. Figure 2 shows typical γ spectra in the case of Ni^{58} and Ni^{60}. By calibrating with a Na^{24} source we were able to determine the absolute efficiency of our γ-ray detector. Taking into account solid angle as well as intrinsic efficiency, the over-all efficiency was typically of the order of 7×10^{-3}. By summing all the counts in the energy interval

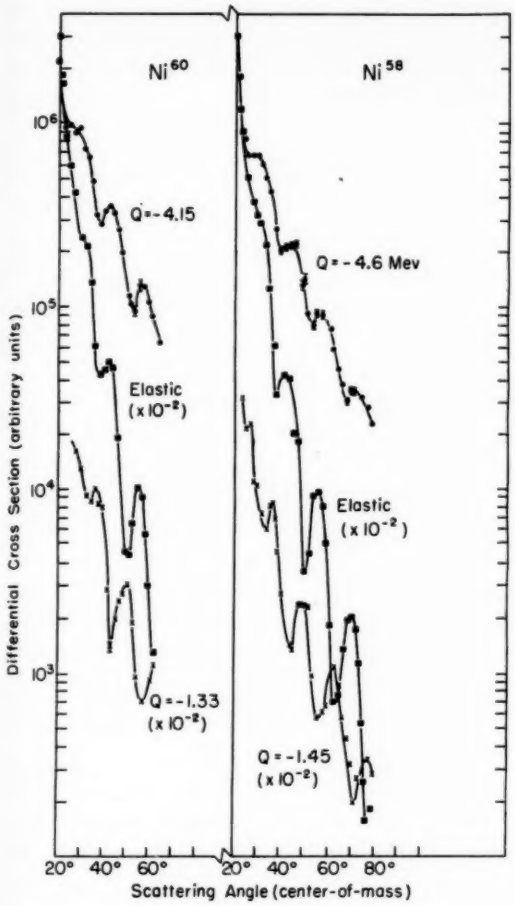

FIG. 1. Angular distribution of alpha particles scattered from Ni^{58} and Ni^{60}.

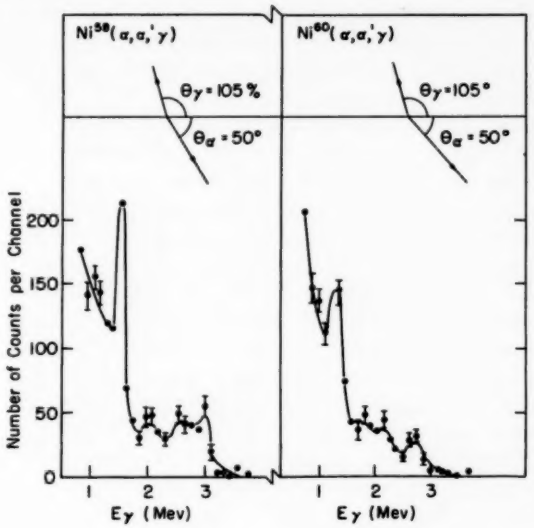

FIG. 2. Gamma spectra from Ni^{58} and Ni^{60} coincident with alpha particles inelastically scattered by the anomalous state.

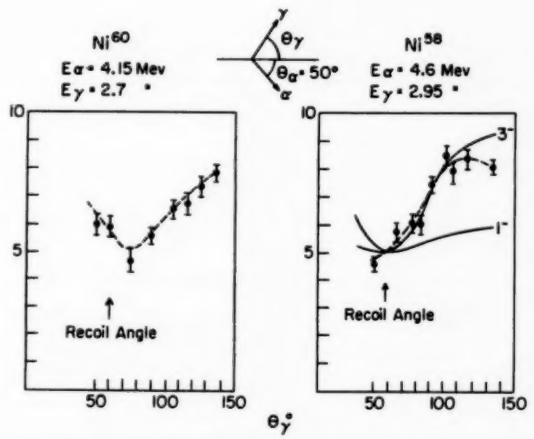

FIG. 3. Alpha-gamma angular correlations from Ni^{58} and Ni^{60}.

from 1.6 to 3.0 Mev in Ni^{60} and 1.85 to 3.35 Mev in Ni^{58}, we were able to show that approximately $(62 \pm 20)\%$ of the time the anomalous structure was excited in Ni^{58} and $(50 \pm 20)\%$ in Ni^{60} the level or levels involved decayed to the first excited state of these nuclei.

Using this same arrangement, the angular correlation of the γ ray from the "anomalous level" to the first excited of each of these nuclei relative to the outgoing alpha particle (and therefore recoiling nucleus) was determined. The results of these measurements are shown in Fig. 3.

The results of the angular distribution measurements can be interpreted as showing that the parity of the "anomalous state" in Ni^{58} and Ni^{60} is different from that of the ground state whereas the parity of the first excited state is the same as that of the ground state. This result follows from either a Born approximation calculation of direct inelastic alpha-particle scattering or the more recent inelastic diffraction calculation of Blair.[4] Inasmuch as both isotopes are even-even, we assume that the ground-state parity is even.

Our results on the first excited states are then in agreement with the known spin and parities of

the first excited states of Ni^{58} and Ni^{60},[5] and give negative parity for the anomalous state. The ambiguity in what value of the radius to use prevents a unique deduction of the spin of the state from the inelastic-scattering angular distribution.[4] On the other hand, the angular correlation results can be interpreted as showing that the "anomalous state" is 3^- and decays to the first excited 2^+ state through the emission of electric dipole radiation[4,6]. The results of a Born approximation calculation for the angular correlation are also illustrated in Fig. 3 for the two assumptions of electric dipole radiation from either a 1^- or 3^- state. By virtue of the fact that the gamma-ray spectrum shows transition directly from the anomalous level to the first and the relatively large number of levels[7] between 1.5 and 4 Mev in both nuclei, it would seem unlikely that the state had any spin higher than 3^-.

A possible interpretation of this level is that it is a collective octupole oscillation.[4,8] On the basis of a classical hydrodynamic approach to nuclear structure Bohr and Mottelson estimate the energy of the lowest octupole surface oscillation to be in the range of 4 to 7 Mev.[9] However, further data relative to the transition probabilities will be necessary to establish this collective nature.[8] Goodman[10] has emphasized the single-particle explanation of the "anomalous levels." While accounting for a level in the inelastic particle spectrum at about 4-Mev excitation, he makes no definite prediction as to its spin.

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EXPERIMENTAL EVIDENCE FOR D WAVES IN π^--p SCATTERING AT 370 AND 427 Mev*

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The differential elastic scattering of pions on protons furnishes information on the magnitude of the various partial waves included in the interaction. Precise measurements of this process up to 333 Mev[1,2] have been interpreted in terms of S and P waves and their six associated charge-independent phase shifts. No D or higher waves were necessary to give a statistically significant fit to the data. In this Letter, we report our values of the elastic-scattering cross section for negative pions on protons at 370 ± 9 and 427 ± 10 Mev (lab energies).

The experimental setup is shown in Fig. 1. The pion beam from an internal Be target in the 184-in. cyclotron was deflected out of the machine by its magnetic fringing field. The beam was then momentum-analyzed and focused by the wedge and quadrupole magnets shown. The intensity of the 2-in. diam beam was greater than 15000 negative pions per second. The beam energy and its muon and electron contaminations were determined from ranges in copper.

The pion beam was scattered in a 4-in.-thick liquid-hydrogen target. The elastically scattered pions were detected by a three-scintillator counter telescope which discriminated against recoil protons and inelastic pions on the basis of range in the copper absorbers placed between

the first and last scintillators of the telescope. A different thickness of copper absorber was used for each energy of the elastically scattered pions. This energy was a unique function of the angle of scattering and was calculated from the kinematics of the two-body final state. The efficiency of the telescope was measured as a function of pion energy, with essentially the same pion beam setup shown in Fig. 1. Beam energy and contamination were determined from range curves.

The main corrections to the data are for Coulomb scattering, finite target and telescope size, and π^0 photons from charge-exchange scattering.[3] The results of the experiment are shown in Fig. 2. Also shown is the differential cross section at zero degrees. The imaginary part of the forward-scattering amplitude was obtained from total cross-section data,[1,4] and the real part was obtained from dispersion theory.[5] Because of the relatively small contribution to the cross section of the real part compared to the imaginary part at these energies, even a 100% error in the real part does not significantly affect the cross-section magnitude or its error. Thus the zero-degree differential cross sections shown depend effectively only upon the experimentally determined total cross sections.

The data were fitted to a power series in $\cos\theta^*$, where θ^* is the center-of-mass scattering angle,

$$d\sigma/d\Omega^* = \sum_{k=0}^{n} A_k(\cos\theta^*)^k,$$

for various values of n. Three of the least-squares fits made at each energy are shown in Fig. 2. The dashed curve represents the fit to the points, excluding the calculated zero-degree point and using only S- and P-wave contributions ($n=2$). The dot-dash curve represents the fit to the same points obtained for a value of n that was

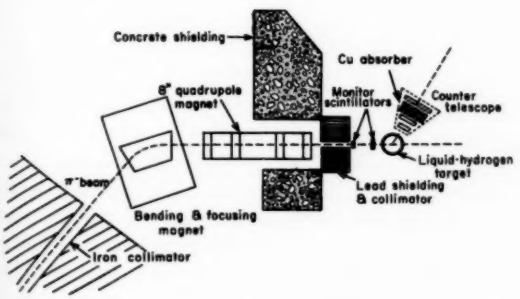

FIG. 1. Experimental arrangement (not to scale).

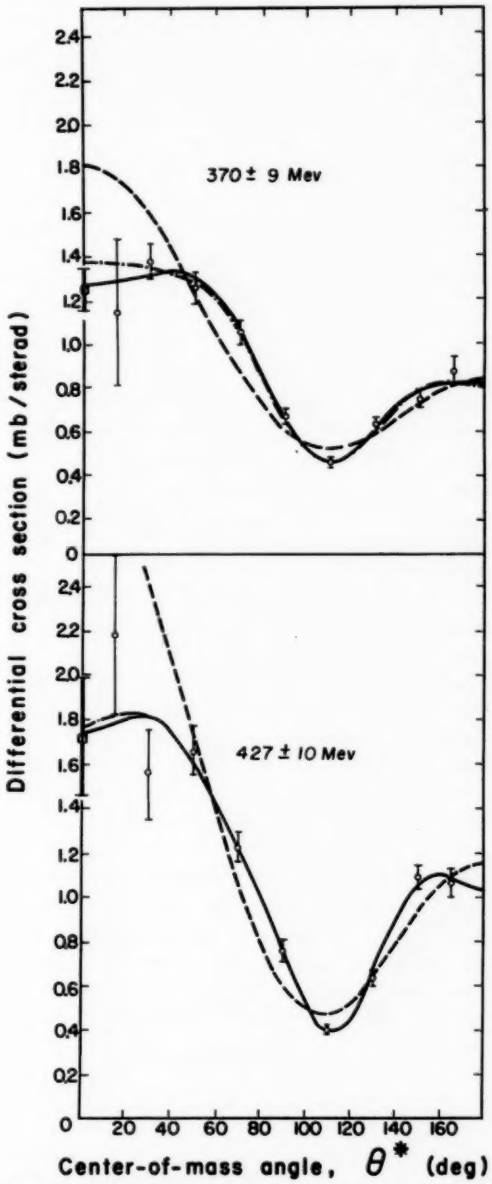

FIG. 2. Differential cross sections for $\pi^- - p$ elastic scattering. The square points on the ordinates represent dispersion-relation predictions. The solid curve represents the best least-squares fit obtained including the zero-degree point. The dot-dash curve represents the best fit obtained excluding the zero-degree point. The dashed curve represents the fit obtained excluding the zero-degree point and using only S and P waves.

Table I. Least-squares-fit parameters, A_k, in millibarns/steradian, for the most probable values of n, for which p is the probability that χ^2 for a random sample would exceed the value found.

Energy (Mev)	n	Zero-deg point	A_0	A_1	A_2	A_3	A_4	A_5	A_6	p (%)
370 ± 9	5	included	0.65 ±0.03	1.02 ±0.10	0.97 ±0.18	-1.43 ±0.40	-0.57 ±0.19	0.65 ±0.35		44
370 ± 9	5	excluded	0.66 ±0.03	1.03 ±0.11	0.89 ±0.25	-1.56 ±0.51	-0.45 ±0.32	0.81 ±0.52		28
427 ± 10	6	included	0.77 ±0.04	1.46 ±0.16	0.19 ±0.56	-2.15 ±0.73	2.50 ±1.68	1.04 ±0.65	-2.07 ±1.26	34
427 ± 10	6	excluded	0.77 ±0.05	1.47 ±0.21	0.20 ±0.70	-2.19 ±1.05	2.43 ±2.20	1.10 ±1.06	-1.99 ±1.78	19

chosen on the basis of a χ^2 test to have its most probable value. The solid curve represents the fit obtained for the most probable value of n when the zero-degree point is included. The values of the best-fit parameters, A_k, are given in Table I. The table also gives the probabilities, p, that χ^2 for a random sample would exceed the value found.

The probabilities that the data are consistent with the fits based on S and P waves alone is less than 1%. Thus the inclusion of D waves is necessary to obtain a reasonable fit to the data. In fact, the most probable values of n obtained include an F-wave interference term ($n = 5$) at 370 Mev and an F-wave term ($n = 6$) at 427 Mev. The conclusion is that D waves are present, and that F waves may be present, especially at 427 Mev.

The total elastic cross sections, obtained from integrating the solid-line fits, are 10.8 ± 0.2 mb at 370 Mev and 13.00 ± 0.34 mb at 427 Mev. The zero-degree differential cross sections obtained by extrapolating the dot-dash curves to 0° are 1.38 ± 0.23 mb/sterad at 370 Mev and 1.78 ± 0.55 mb/sterad at 427 Mev. Within statistics these values agree with those predicted from dispersion relations.

We should like to acknowledge the continued interest and support of Professor A. C. Helmholz, and to thank Dr. Walton A. Perkins and Mr. James Vale and the cyclotron crew for their assistance and cooperation during the course of the experiment.

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PARITY OF THE NEUTRAL PION*

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The parity of the pion relative to the nucleon is essential to the understanding of the strong interactions. Our most direct knowledge, to date, comes from the observation of the capture reaction $\pi^- + d \rightarrow 2n$.[1] It can be shown that this capture proceeds from the s atomic state,[2] and for scalar pions s-state capture would be forbidden. The neutral pion parity can then be inferred either from the mounting evidence on the validity of charge independence, or perhaps more directly from the forbiddenness of the reaction $\pi^- + d \rightarrow \pi^0 + 2n$.[3]

The possibility of a direct determination of the neutral pion parity, using the decay $\pi^0 \rightarrow 2\gamma$, was pointed out by Yang quite some time ago.[4] Yang demonstrated that angular momentum and parity conservation require parallel and perpendicular correlation of the polarizations of the two photons, for scalar and pseudoscalar pions, respectively. The experiment has not been carried out so far, however, because no one has solved the problem of the measurement of the relative polarizations of gamma rays of this energy (70 Mev).

Here we present a preliminary report of a study of certain angular correlations in the decay $\pi^0 \rightarrow e^+ + e^- + e^+ + e^-$. This decay is expected theoretically, and is calculable on the basis of quantum electrodynamics. It is the internal conversion of the two photons of the normal decay. The planes of the pairs "remember" the polarization of the virtual intermediate photons; the correlation of these planes reflects then the polarization correlation of Yang. The correlation of the planes in double internal conversion has been calculated by Kroll and Wada.[5]

Since the parity is not inferred from the observed correlations on the basis of invariance arguments alone, but is linked to them through the dynamical theory of internal conversion, it is important that the theory be thoroughly reliable. To this end we point out, that in addition to the success of quantum electrodynamics in previous applications, it has now been tested in the details of both single and double internal pair creation of π^0 decay (absolute rates, energy, and angular distributions)[6] and has been confirmed in every instance within experimental limits.

The experimental difficulty is due to the rarity of the decay (1/30 000) in combination with the necessity of establishing at least the relative orientation of the planes of the two pairs. In this experiment, the π^0's are produced in the capture reaction $\pi^- + p \rightarrow \pi^0 + n$. About 60% of the negative pions coming to rest in hydrogen undergo this reaction, emitting 4-Mev π^0 mesons. A liquid hydrogen bubble chamber, 30 cm diam, 15 cm deep, in a field of 5500 gauss, is exposed to a stopping π^- beam. 700 000 pictures have been taken, and there are ~15 stopping mesons per picture. We expect therefore that ~200 events will be found eventually. We report here an analysis of the first 103 events, since we believe that this already permits a convincing conclusion.[7] Figure 1 exhibits a photograph of a typical double internal conversion.

In each event the momenta and directions of the 4 tracks were measured, and transformed to the center-of-mass system of the decaying π^0. Nine of the events were not measurable. In addition, 15 events in which the measured angle between any pair was less than 3 degrees were excluded, on the grounds that the plane normal was inadequately measured. The remaining 79 events should now be analyzed with the help of the theoretical result of Rockmore.[8] We hope to do this eventually, but it encounters some technical difficulties which we are still fighting. Here we make the analysis with the help of the theory

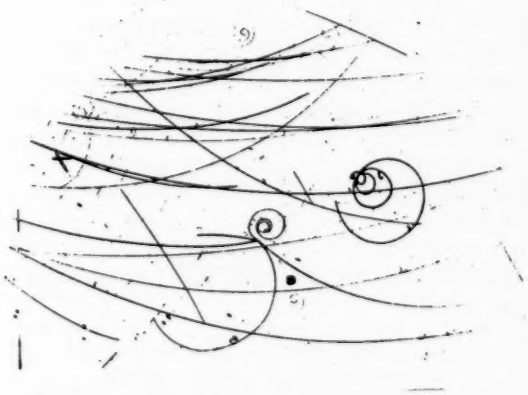

FIG. 1. A photograph of a typical double internal conversion.

which neglects exchange.[5] It is necessary to give a short account of this neglect. If 1, 2 are the positrons, 3, 4 the electrons, two pairings are possible: (13), (24) or (14), (23). The decay rate will then be of the form $\{|M[(13)\ (24)]|^2 + |M[(14)\ (23)]|^2 + 2\ \mathrm{Re}M^*[(13)\ (24)]M[(14)\ (23)]\}$. The formula of Kroll and Wada[5] consists of the first or second term, depending on how the pairing is chosen. Usually the angles of the pairs are quite small (~10°) so that the proper pairing is clear on sight. Theoretically small angles have the consequence that one of the first two terms is much larger than the other. If this pairing is used, then it is clear that the terms neglected in Kroll and Wada are small. We have adopted the criterion $|M[(13)\ (24)]|^2 > 4|M[(14)\ (23)]|^2$ for the applicability of the Kroll-Wada formula. Fifteen events failed to qualify; so that 64 events remain.

Let ϕ be the angle between the planes of the two pairs. The theoretical correlation is then of the form

$$E_s(\phi) = 1 + \alpha(x_1, y_1; x_2, y_2)\ \cos 2\phi,$$

$$E_{ps}(\phi) = 1 - \alpha(x_1, y_1;\ x_2, y_2)\ \cos 2\phi,$$

where the correlation coefficient $\alpha(x_1, y_1;\ x_2, y_2)$ is the function of the remaining four nontrivial observables, which can be taken to be the angle and energy division within each pair. The parity dependence of F is entirely in the sign. α is calculated for each event. Its square is a measure of the usefulness of the event, and varies widely from event to event.

We have tabulated the results in two ways. In the first case we plot the observed distribution in ϕ, weighting each event with the square of its correlation coefficient α. In this way the graph of Fig. 2 is obtained. It can be analyzed for an effective α, $\bar{\alpha}$. We find $\bar{\alpha}_{\mathrm{exp}} = -0.75 \pm 0.42$. The theoretically expected value for our distribution in $\alpha(x_1, y_1;\ x_2, y_2)$ is $\alpha_{\mathrm{th}} = \pm 0.48$, where the plus and minus sign refer to even and odd parity, respectively. The alternate method is that of relative likelihood. The relative likelihood $L(ps)/L(s)$ is obtained by finding for each event the relative probability for finding the event on the basis of pseudoscalar versus scalar theory, and forming the product

$$\frac{L(ps)}{L(s)} = \prod_{i=1}^{64} \frac{1-\alpha_i \cos 2\phi_i}{1+\alpha_i \cos 2\phi_i} = 1500.$$

(If the events excluded for the above-stated reasons were included, it would increase the relative likelihood to a figure of about 30 000.) Both types of analysis lead to the conclusion that the neutral pion is pseudoscalar. In each case, the confidence in the conclusion corresponds to about three standard deviations.

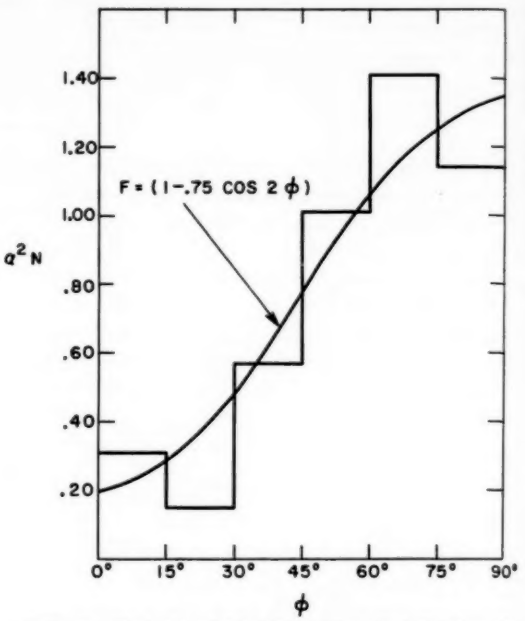

FIG. 2. Plot of weighted frequency distribution of angle between planes of polarization.

This to us seems statistically quite convincing. It is fortunate that systematic errors in correlations of the planes would be hard to achieve. We can state with confidence that no such errors, sufficient to influence the result, are present. It can then be concluded that the neutral pion is pseudoscalar. The above-quoted arguments can then be inverted to argue that also the charged pion is pseudoscalar, in agreement with the earlier results.[1]

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TRANSFORMATION OF MUONS INTO ELECTRONS*

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It is an outstanding puzzle of particle physics that muons do not decay electromagnetically into electrons without the emission of neutrino pairs, even though all quantum numbers of muon and electron are the same.[1] Processes of this sort which could have been observed include $\mu \to e + \gamma$ decay,[2] $\mu \to e$ via absorption of virtual photons in a mesonic atom,[3] $\mu \to 3e$ via internal conversion,[4] muonium decay into photons,[5] etc. The absence of such transformations does not constitute a paradox, there being no compelling reason why muons should transform into electrons, but it seems a mystery that processes which are allowed energetically and in every other known respect do not occur. In this note we shall indicate how for a certain class of interactions which might be expected to induce μ-e transformations, the similarity of the μ and e actually forbids the transformations.

Let us first consider why $\mu \to e$ transformations do not take place as strong and electromagnetic interactions; for example, why is $\mu \to e + \gamma$ decay not about as fast as $\Sigma^0 \to \Lambda^0 + \gamma$ decay? Let us suppose that the muon and electron are described by a local Lagrangian $\mathcal{L}$ involving only the fields $\psi_e, \psi_\mu, A^\lambda$ where ψ_e and ψ_μ can be chosen to have the same parity. Now, the simplest mechanisms that might yield fast $\mu \to e$ transformations are contact terms in $\mathcal{L}$,

$$\mathcal{L}_1 = -\rho\bar{\psi}_e\psi_\mu - \text{H.c.}; \quad \mathcal{L}_2 = -\xi\bar{\psi}_e\gamma\cdot D\psi_\mu - \text{H.c.}, \qquad (1)$$

where the derivative ∂_λ and the photon field A_λ enter in the gauge-invariant combination, $D_\lambda = \partial_\lambda - ieA_\lambda$. These two interactions exhaust all possible parity-conserving, renormalizable, gauge- and Lorentz-invariant interactions among photons, muons, and electrons that can be added to the usual Dirac Lagrangian in order to generate $\mu \to e$ transformations. Furthermore, because the μ and e appear to differ only in their rest mass, it seems not unreasonable to regard them as "different states of the same particle." In this case, since the bare Lagrangian is known to contain $\bar{\psi}_\mu\psi_\mu$ and $\bar{\psi}_e\psi_e$ terms of the type 1, one might also expect "off-diagonal" $\bar{\psi}_\mu\psi_e$ terms. However, since quantum electrodynamics has been successful without the introduction of nonminimal electromagnetic interactions or other nonrenormalizable interactions, one is also inclined to exclude "off-diagonal" terms of this kind, at least from the fundamental Lagrangian.

Suppose we now attempt to calculate, as an example, the lowest order μ-e-γ vertex. There are five diagrams that add to give a nonvanishing

function of the lepton momenta,

$$\Gamma_{e\mu}{}^{\lambda} = e\rho[\gamma^{\lambda}S(p_{\mu}, m_e) + S(p_e, m_{\mu})\gamma^{\lambda}] + e\xi[\gamma^{\lambda} + \gamma^{\lambda}S(p_{\mu}, m_e)ip_{\mu}\cdot\gamma + ip_e\cdot\gamma S(p_e, m_{\mu})\gamma^{\lambda}], \quad (2)$$

where $S(p, m) = (i\gamma\cdot p - m)/(p^2 + m^2)$. However, when we put the lepton momenta on their mass shells (i.e., set $p_{\mu}\cdot\gamma = im_{\mu}$, $p_e\cdot\gamma = im_e$), we find a striking cancellation; the sum of the first two, and of the last three terms becomes zero.

We shall now show that the cancellation of the matrix element for $\mu \to e$ transformations takes place to <u>all</u> orders in ρ, ξ and e provided that $|\xi| < 1$, and with any number of real or virtual photons emitted or absorbed. Thus it is impossible to induce $\mu \to e$ transformations without introducing either new fields, or more complicated (and nonrenormalizable) interactions such as

$$\bar{\psi}_e \sigma_{\lambda\eta} \psi_{\mu} F^{\lambda\eta}.$$

Our proof is based on the fact that there exists a new set of fields, $\psi_{e'}, \psi_{\mu'}$ which are linear combinations of ψ_e and ψ_{μ} such that the total Lagrangian when written in terms of the ψ' becomes just the sum of two uncoupled Dirac Lagrangians for $\psi_{e'}$ and $\psi_{\mu'}$ separately. To see this let us introduce a matrix notation, writing

$$\psi = \begin{pmatrix} \psi_{\mu} \\ \psi_e \end{pmatrix}.$$

Then the Lagrangian obtained by adding $\mathfrak{L}_1 + \mathfrak{L}_2$ to the usual Dirac Lagrangian may be written

$$\mathfrak{L} = -\bar{\psi}\gamma\cdot D\mathfrak{A}\psi - \bar{\psi}\mathfrak{B}\psi + \mathfrak{L}_m, \quad (3)$$

where $\mathfrak{L}_m = -\frac{1}{4}F_{\lambda\eta}F^{\lambda\eta}$, and

$$\mathfrak{A} = \begin{pmatrix} 1 & \xi^* \\ \xi & 1 \end{pmatrix}, \quad \mathfrak{B} = \begin{pmatrix} m_{\mu} & \rho^* \\ \rho & m_e \end{pmatrix}. \quad (4)$$

We shall only make use of the Hermiticity of $\mathfrak{A}$ and $\mathfrak{B}$, and the positivity of $\mathfrak{A}$ (for $|\xi| < 1$); our proof will work for any number of fields. The condition that $\mathfrak{A}$ is positive is required to make the canonical anticommutation relations consistent and so is not really a new assumption. If we introduce the fields ψ' by writing

$$\psi = S\psi', \quad (\mathrm{Det}S \neq 0) \quad (5)$$

then clearly $\mathfrak{L}$ may be rewritten as

$$\mathfrak{L} = -\bar{\psi}'\gamma\cdot DS^{\dagger}\mathfrak{A}S\psi' - \bar{\psi}'S^{\dagger}\mathfrak{B}S\psi' + \mathfrak{L}_m. \quad (6)$$

It is well known[6] that even though $\mathfrak{A}$ and $\mathfrak{B}$ may not commute, we can always choose S such that $S^{\dagger}\mathfrak{A}S = 1$ and $S^{\dagger}\mathfrak{B}S$ is diagonal. When we choose S in this fashion, $\mathfrak{L}$ becomes

$$\mathfrak{L} = -\bar{\psi}_{e'}(\gamma\cdot D + m_{e'})\psi_{e'} - \bar{\psi}_{\mu'}(\gamma\cdot D + m_{\mu'})\psi_{\mu'} + \mathfrak{L}_m, \quad (7)$$

where $m_{e'}$ and $m_{\mu'}$ are given by the eigenvalues of $S^{\dagger}\mathfrak{B}S$. The field equations and commutation rules for ψ' are the usual equations for uncoupled leptons, whether deduced from (7) or from (3) and (5).

By inspection we see that the theory described by (7) possesses <u>two</u> stable leptons, which of course we call electron and muon, and is invariant under separate gauge transformations on $\psi_{e'}$ and $\psi_{\mu'}$, which means that transitions between e' and μ' are forbidden. However, it may not be clear why we obtain a cancellation when [as in (2)] we compute the Feynman diagrams for $\mu \to e$ transformations from (3), rather than the $\mu' \to e'$ transformations. When we compute using (3), what we obtain is the Fourier transform of a quantity

$$\Gamma_{iJ}{}^{\lambda_1\cdots\lambda_n}(x, y, z_1, \cdots z_n) = \langle T\{\psi_i(x)\bar{\psi}_J(y), A^{\lambda_1}(z_1)\cdots A^{\lambda_n}(z_n)\}\rangle_0, \quad (i, J = \mu, e), \quad (8)$$

whereas using (7) we obtain a quantity Γ_{iJ}' defined correspondingly in terms of ψ_i' and ψ_J'. Using (5) we have

$$\Gamma = S\Gamma' S^{\dagger}, \quad (9)$$

and since $\Gamma_{e\mu'}=0$,

$$\Gamma_{e\mu}=S_{ee}S_{\mu e}{}^{*}\Gamma_{ee'}+S_{e\mu}S_{\mu\mu}{}^{*}\Gamma_{\mu\mu'}, \tag{10}$$

and, as in (3), this is not zero. But when we compute matrix elements from (10), what we calculate is[7]

$$\int d^4x d^4y\, \bar{u}_e(x)\left[\left(\gamma\cdot\frac{\partial}{\partial x}+m_e{}^R\right)\Gamma_{e\mu}{}^{\lambda_1\cdots\lambda_n}(x,y,z_1,\cdots z_n)\left(\gamma\cdot\frac{\partial}{\partial y}+m_\mu{}^R\right)\right]u_\mu(y), \tag{11}$$

where $m_e{}^R$, $m_\mu{}^R$ are renormalized masses. Substitution of (10) into (11) gives zero for both terms, since $\Gamma_{ee'}$ has no singularity on the muon mass shell, so that we can integrate by parts and use

$$\left(\gamma\cdot\frac{\partial}{\partial y}+m_\mu{}^R\right)u_\mu(y)=0, \tag{12}$$

and similarly $\Gamma_{\mu\mu'}$ has no singularity on the electron mass shell. Actually it may be seen from a very general theorem[8] on the particle interpretation of field theory that the matrix element $\langle e|T\{A^{\lambda_1}\cdots A^{\lambda_n}\}|\mu\rangle$ is given correctly in general (except for field renormalization constants) by applying the reduction formula (11) to either $\Gamma_{e\mu'}$ or $\Gamma_{e\mu}$ or even to the diagonal elements of Γ' or Γ.

If the μ and e had anomalous magnetic moments of nonelectromagnetic origin, the theorem would not in general hold, and the existence of the terms (1) would then lead to μ-e transitions.

If the original Lagrangian contained weak interactions, given for example (symbolically) by

$$\mathfrak{L}_\beta=-\bar{\psi}_\nu O\psi_i\bar{\psi}_n O\psi_p\beta_i-\text{H.c.}, \tag{13}$$

$$\mathfrak{L}_{\bar{\nu}\nu}=-\bar{\psi}_J O\psi_i\bar{\psi}_\nu O\psi_\nu n_{Ji}, \tag{14}$$

then $\mathfrak{L}_\beta$ and $\mathfrak{L}_{\bar{\nu}\nu}$ can also be written in the same way, in terms of ψ',β',n', where

$$\beta'=\beta S,\quad n'=S^{\dagger}nS. \tag{15}$$

If the μ and e had other weak Fermi-type interactions such as $\bar{\psi}_\mu\psi_\mu\bar{\psi}_e\psi_e$, the transformation to the new fields μ',e' would induce terms which give $\mu'\to e'$ transitions. These would be comparable in strength to ordinary weak interactions.

The above discussion indicates that we may be able to define "fundamental" lepton fields ψ such that (a) the free fields and electromagnetic interactions are invariant under the interchange of ψ_e and ψ_μ; (b) the weak interactions break this symmetry by involving only one (say ψ_μ) of these two fields; (c) there are no off-diagonal derivative-type interactions. This is accomplished by setting

$$\xi=0,\quad \rho\ \text{real},\quad m_e=m_\mu=m,$$

$$\beta\simeq(1,0),\quad \eta\simeq\begin{pmatrix}1&0\\0&0\end{pmatrix}. \tag{16}$$

It then follows that the matrix S is independent of ρ and m, and given by

$$S=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}, \tag{17}$$

so that the experimentally observable parameters become

$$\beta'\sim\frac{1}{\sqrt{2}}(1,1),\quad n'=\begin{pmatrix}1&1\\1&1\end{pmatrix}\frac{1}{2}. \tag{18}$$

Thus this hypothesis is in agreement with the experimental fact that β decay and μ absorption have about the same unrenormalized coupling constants. (We can also obtain $m_e'\ll m_\mu'$ by assuming $\rho\simeq m$.) It is in any case possible to define ψ_e and ψ_μ so that the asymmetry between them is induced by weak interactions and not by the free field, rather than vice versa, provided that the original Lagrangian is of the form (4) + (13) + (14).

We may also apply these considerations to the case of weak contact terms, diagonal or off-diagonal. Here we may not assume invariance under space inversion, and hence must also introduce terms of form

$$\mathfrak{L}_3\sim\bar{\psi}\gamma_5\mathfrak{D}\psi+\bar{\psi}D\cdot\gamma\gamma_5\mathfrak{C}\psi. \tag{19}$$

The total Lagrangian is then

$$\mathfrak{L}=-\bar{\psi}[D\cdot\gamma\mathfrak{A}+\mathfrak{B}+D\cdot\gamma\gamma_5\mathfrak{C}-i\gamma_5\mathfrak{D}]\psi+\mathfrak{L}_m, \tag{20}$$

where $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$ are Hermitian and differ from their values in (3) by terms of first order in the weak interactions. It can be shown that, defin-

ing ψ' by

$$\psi = (S + T\gamma_5)\psi', \quad [\mathrm{Det}(S + T\gamma_5) \neq 0], \tag{21}$$

we can choose S and T such that $\mathfrak{L}$ is given by (7), to first order in the weak interactions.[9] This is of some "practical" importance, since if there is any structure in μ decay,[10] it may lead to effective interactions such as those discussed above through diagrams in which ν and $\bar{\nu}$ annihilate virtually. But the terms of form (1) and (20) cannot themselves lead to any electromagnetic $\mu \rightarrow e$ transitions.[11]

We would like to thank Dr. M. Goldhaber for interesting discussions about the μ-e system.

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[1]Unless we are willing to give up the 2-component neutrino theory, we know that $\mu \rightarrow e + \nu + \bar{\nu}$.

[2]G. Feinberg, Phys. Rev. 110, 1482 (1958); Berley, Lee, and Bardon, Phys. Rev. Letters 2, 357 (1958).

[3]S. Weinberg and G. Feinberg, Phys. Rev. Letters 3, 111, 244 (1959); J. Steinberger and H. Wolfe, Phys. Rev. 100, 1480 (1955).

[4]Juliet Lee and N. P. Samios, Phys. Rev. Letters 3, 55 (1959).

[5]York, Kim, and Kernan (to be published).

[6]H. Goldstein, Classical Mechanics (Addison-Wesley Publishing Company, Cambridge, 1950), p. 323.

[7]Lehmann, Symanzik, and Zimmermann, Nuovo cimento 2, 425 (1955).

[8]R. Haag, Phys. Rev. 112, 669 (1958).

[9]It is likely that this theorem is true to all orders in "weak interactions" of the type included in the Lagrangian (20). That is, by a general transformation of the form (21), one can transform $\mathfrak{L}$ into a new Lagrangian in which $\mathfrak{A} = 1$ and $\mathfrak{B} = \begin{pmatrix} m_\mu' & 0 \\ 0 & m_e \end{pmatrix}$, with $\mathfrak{C} = 0$ and $\mathfrak{D} = 0$.

[10]S. Bludman and A. Klein, Phys. Rev. 108, 550 (1958); T. D. Lee and C. N. Yang, Phys. Rev. 108, 1611 (1958).

[11]This fact has been noted by M. Gell-Mann, 1958 Annual International Conference on High-Energy Physics at CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 261, and by N. Cabibbo and R. Gatto (to be published). The proof of Cabibbo and Gatto actually applies directly only to the nonderivative terms of (1) and (20). A very simple general proof is as follows: We have already shown that diagrams containing vertices of type (1) and (2) alone, or (20) and (21) alone, do not lead to $\mu \rightarrow e$ transformations. But to lowest order in weak interactions, each diagram contains a single vertex of type (1) or (20).

ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

VELOCITY OF SOUND IN LIQUID He^3 AT HIGH PRESSURES. K. R. Atkins and H. Flicker, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received July 14, 1959).

The velocity of 14-Mc/sec sound in liquid He^3 was measured at pressures up to 9 atmospheres in the temperature range 1.2°K to 3.2°K. A few measurements were also made in the gas just above the critical temperature. A possible phonon contribution to the thermal coefficient of expansion of the liquid is discussed.

INCOHERENT MICROWAVE RADIATION FROM PLASMAS. G. Bekefi, Jay L. Hirshfield, and Sanborn C. Brown, Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received July 6, 1959).

A study is made of the incoherent radiation from an isotropic, quiescent plasma of a low degree of ionization. Three cases are treated theoretically: the transparent plasma, the semiopaque plasma, and the opaque plasma. Radiation from positive columns of dc glow discharges in helium and hydrogen for the three cases treated theoretically is studied experimentally at 3000 Mc/sec, and good agreement is obtained between theory and experiment.

SCATTERING POTENTIAL IN FULLY IONIZED GASES. O. Theimer and R. Gentry, Department of Physics, University of Oklahoma, Norman, Oklahoma (Received June 29, 1959).

Recent methods for calculating the probability $W(E)dE$ that a test particle in a plasma experiences an electrical field of magnitude $E \pm \frac{1}{2}dE$ are discussed. In particular, the assumption is analyzed that the distribution $W(E)$ in a system of true charges can be calculated from an auxiliary system of uncorrelated virtual particles which, formally, do not interact although they are the sources of shielded Coulomb fields of the type introduced by Debye and Hückel. It is shown that the electrostatic energy of the auxiliary system, calculated as a volume integral over the average energy density $\langle E^2 \rangle_{Av}/8\pi$, is identical with the interaction energy of the true particles as obtained by the Debye-Hückel theory. The distribution of the micropotential $W(\psi)d\psi$ in a plasma is also calculated from the auxiliary system and used for deducing a scattering potential $\psi(r)$ which has a natural cutoff at a distance equal to the average distance between neighboring ions. An approximate analytical expression for $\psi(r)$ is given and the physical nature of the scattering potential is discussed.

HIGH-CURRENT SUPERCONDUCTIVITY. R. H. Parmenter, RCA Laboratories, Princeton, New Jersey (Received July 9, 1959).

The Bardeen, Cooper, Schrieffer theory of superconductivity is extended to the case of high current densities by explicitly including in the phonon-induced electron-electron attraction the modification of the phonon spectrum in a moving coordinate system. This modification results from the Doppler effect. In materials having cross-sectional dimensions of the order of the penetration depth, it is shown that at current densities of the order of 10^9 amp/cm^2 the superconducting state is stable relative to the normal state, this holding both in superconducting metals and in metals ordinarily not considered superconducting at all. For this high-current state, it appears possible to obtain superconducting transition temperatures as high as room temperature. Means for achieving such an effect experimentally are discussed.

DEPENDENCE OF THE FORBIDDEN GAP AND LUMINESCENCE GROUND-STATE ENERGIES OF (ZnCd)S:Ag ON THE CONCENTRATION OF CdS. Gordon E. Gross, Midwest Research Institute, Kansas City, Missouri (Received July 6, 1959).

A series of (ZnCd)S:Ag:Cl phosphors have been studied to determine the effect of increased CdS content on the forbidden gap and on the energy of the ground states associated with luminescence.

This work is correlated with that of other workers to confirm the presence of two separate ground states and to show that the luminescence of this material may be adequately explained by the Klasens model. Also the uniform translation of the spectral peaks (while retaining their shape) emphasizes the long-range order aspects of these phosphors.

LOW-TEMPERATURE ANNEALING OF IRRADIATION-INDUCED DEFECTS IN LiF. D. A. Wiegand and R. Smoluchowski, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received July 15, 1959).

The expansion of alkali halides due to x-ray irradiation has been observed at 90° and 300°K. Simultaneous measurements of the *F* band indicate that the expansion can be roughly explained on the basis of the generation of negative-ion vacancies. However, partial annealing of the expansion of LiF at approximately 130°K after irradiation at 90°K is not accompanied by a decrease in the density of negative-ion vacancies associated with *F* centers, therefore indicating that the expansion cannot be completely due to the increase in the density of *F* centers. The low-temperature annealing of the expansion can be associated with either vacancy-pair diffusion or interstitial motion. While experiments with plastic deformation and *M*-band formation in LiF argue against vacancy-pair diffusion, the spin resonance experiments of Känzig and Woodruff support the interstitial interpretation. It is therefore tentatively concluded that halogen vacancy-interstitial pairs are generated by x-ray irradiation in LiF at 90°K and that the low-temperature annealing is due to a change in interstitial configuration.

RELAXATION EFFECTS IN A MASER MATERIAL, $K_3(CoCr)(CN)_6$. S. Shapiro* and N. Bloembergen, Gordon McKay Laboratory, Harvard University, Cambridge, Massachusetts (Received July 6, 1959).

The rate equations for the occupation of spin levels are augmented to include cross-relaxation processes. It is confirmed experimentally that the latter are important when the concentration of magnetic ions is high or two resonances have a small separation. Maser action is usually impaired under such circumstances. When experimental conditions are chosen such that cross-relaxation effects are negligible, it is shown that all spin-lattice relaxation processes for Cr^{+++} in the cyanide are proportional to the absolute temperature in the liquid helium range. The susceptibilities at both the pump and maser frequencies follow the general theoretical dependence and reach asymptotic values as a function of pump power.

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RESOLUTION OF THE AMBIGUITY OF VALENCE STATES IN SPINELS CONTAINING MANGANESE AND IRON. Arthur Miller, Radio Corporation of American Laboratories, Princeton, New Jersey (Received July 7, 1959).

The dilemma of whether $[Fe^{+3}Mn^{+2}]$ or $[Fe^{+2}Mn^{+3}]$ are the stably coexisting species on the octahedrally ligated sites of the spinel structure has been resolved by the application of a crystallographic, rather than a magnetic criterion. The solid solution systems,

$$(Zn[Mn_2]O_4)_{1-x}\cdot(Zn_{0.5}Ge_{0.5}[FeMn]O_4)_x$$

and

$$(Zn[FeMn]O_4)_x\cdot(Zn_{0.5}Ge_{0.5}[Mn_2]O_4)_{1-x},$$

were synthesized, and their lattice constants were determined. The compositions in these systems for which cooperative tetragonal distortion occurs, due to the Jahn-Teller effect for Mn^{+3}, depend on whether $[Fe^{+3}Mn^{+2}]$ or $[Fe^{+2}Mn^{+3}]$ are stably coexisting species. The crystallographic findings are in complete agreement with the latter valence assignment. Furthermore, the axial ratios are found to be in quantitative agreement with the theory of cooperative distortions advanced by Wojtowicz, provided that the valence assignment $[Fe^{+2}Mn^{+3}]$ is assumed. The valence behavior observed in the systems studied probably obtains for most other spinels in which iron and manganese coexist on the octahedrally ligated sites.

Unlike magnetic criteria which have been proposed to clarify the valence assignment, the method reported is not subject to ambiguities such as the existence or nonexistence of complex ferrimagnetic coupling schemes (e.g., Yafet-Kittel angular coupling) and "spin quenching."

EXCITON STATES IN IONIC CRYSTALS. R. S. Knox and N. Inchauspé, Department of Physics, University of Illinois, Urbana, Illinois (Received May 25, 1959; revised manuscript received August 7, 1959).

Exciton states in ionic crystals are analyzed according to configurations allowed by cubic point symmetry. The "excitation" and "electron transfer" models of the exciton structure are reintroduced as two slightly different aspects of the same general group-theoretical problem. Predictions of these models concerning multiplicity of absorption peaks are shown to be essentially identical. A theory of the "halogen atom doublet" appearing in the experimental absorption spectra of certain alkali halide crystals is given and is used in a preliminary interpretation of the electronic structure of low-lying exciton states in these solids.

ULTRAVIOLET ABSORPTION OF ALKALI HALIDES. J. E. Eby,* K. J. Teegarden, and D. B. Dutton, Institute of Optics, University of Rochester, Rochester, New York (Received May 27, 1959; revised manuscript received August 19, 1959).

The absorption spectra of evaporated films of the alkali halides, with the exception of LiF, have been measured at room temperature and 80°K in the region from 1100 A to 2500 A. The spectra are rather complex at low temperature, but agree fairly well with current theory concerning the multiplicity and grouping of the exciton peaks. In addition, a shoulder has been found in many of the spectra, similar to the ones previously seen and associated with the band-to-band transition in the alkali iodides.

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X-RAY EXPANSION AND COLORATION OF UNDOPED AND IMPURITY-DOPED NaCl CRYSTALS. Herbert Rabin, United States Naval Research Laboratory, Washington, D. C. (Received July 10, 1959).

Expansion and coloration measurements of undoped and impurity-doped single crystals of NaCl have been made under x-ray irradiation (43 kvp and 20 ma) for intervals up to three hours. Expansion was measured in a direction normal to the x-ray beam with a sensitive capacitive dilatometer that was designed to compensate automatically for thermal fluctuations. It was found that the initial rate of creation of F centers exceeds the rate of creation of vacancy pairs as computed from the expansion data; however, after continued irradiation the rates of F-center and vacancy-pair production are approximately evenly matched. This suggests that the initial production of F centers results from both vacancies originally contained in the crystal and vacancies created during irradiation, while the later coloration is due essentially to vacancies that are created in the irradiation process. Within the framework of this interpretation it is possible to estimate the initial negative-ion vacancy concentration of a number of crystals. This concentration ranges from $1\times10^{16}/cm^3$ for a Harshaw crystal to a value larger than $6\times10^{17}/cm^3$ for a calcium-doped crystal grown at NRL. Although the presence of a large initial concentration of negative-ion vacancies in calcium-doped NaCl explains its enhanced color sensitivity to x-rays without corresponding enhancement of x-ray induced expansion, this explanation is not in accord with simple expectations of charge compensation and the law of mass action.

ELECTROLUMINESCENCE AT LOW VOLTAGES. W. A. Thornton, Lamp Division, Westinghouse Electric Corporation, Bloomfield, New Jersey (Received June 24, 1959).

Electroluminescence occurs in activated ZnS thin films at 1.5 volts rms, corresponding to peak voltage (2.2 volts) less than the band gap (3.8 volts) and less than the mean energy (2.6 volts) of the photons emitted. The light emission decreases by 10^4 between 2.0 and 1.5 volts rms and shows no tendency toward a threshold, nor does its spectral character change at low voltage. This behavior suggests that electroluminescence does not depend upon collision ionization but perhaps on carrier injection or free electron temperature.

ANTIFERROMAGNETIC LINEAR CHAIN. L. R. Walker, Bell Telephone Laboratories, Murray Hill, New Jersey (Received June 4, 1959).

Orbach's integral equation which leads to the value of the ground-state energy of an anisotropic

antiferromagnetic linear chain of spins, $S = 1/2$, has been solved. The result is then expanded in powers of the anisotropy parameter. In this form it corresponds to the result of a perturbation calculation, the transverse part of the Hamiltonian being the perturbation. The rapid convergence of the energy series even for the isotropic case and the adequate convergence of that for the short-range order, suggests that the result given by perturbation theory for the sublattice magnetization may also be satisfactory. It is argued that it is reasonable to expect such a finite sublattice magnetization in the linear chain.

PARAMAGNETIC RESONANCE AND CRYSTAL FIELD IN TWO NICKEL CHELATE CRYSTALS. Martin Peter, Bell Telephone Laboratories, Murray Hill, New Jersey (Received July 2, 1959; revised manuscript received September 8, 1959).

The splitting of the paramagnetic ground state of Ni^{++} in a cubic field perturbed by an axial and an orthorhombic term is calculated. The crystal field can be approximated by the electrostatic field of the surrounding ions, but the close contact of these ions with the impurity makes this an uncertain approximation. We have, therefore, measured the paramagnetic spectrum of Ni^{++} in a metal-organic crystal where the orthorhombic component of the crystal field is caused entirely by two far-removed dioxane molecules. (Upon replacement of the dioxane by benzene, this component disappears.) A comparison between predicted and measured values is given.

RADIATION EFFECTS IN SILICA AT LOW TEMPERATURES. George W. Arnold and W. Dale Compton, United States Naval Research Laboratory, Washington, D. C. (Received June 26, 1959).

Optical absorption bands induced in fused silica and crystalline α-quartz of low impurity content at 77°K by fast electrons or x-rays bleach slowly at room temperature. The presence of OH^- ions in fused silica inhibits the formation of such radiation-induced absorption. Comparison of the number of centers produced at 215 mμ (C band) in Corning 7943 fused silica (OH^- free) for equal absorbed dose when irradiated with electrons and x-rays indicates that displacements are not involved in the initial formation of the color centers. A defect model requiring simple ionization seems adequate to explain most of the observed phenomena in this pure fused silica. No simple model can be proposed which adequately describes the data in the case of the Corning 7940 fused silica (OH^- bearing).

THERMOLUMINESCENCE OF X-RAY-COLORED NaCl CRYSTALS. A. Halperin, N. Kristianpoller, and A. Ben-Zvi, Department of Physics, The Hebrew University, Jerusalem, Israel (Received June 23, 1959).

Glow curves and thermoluminescence spectra taken at various temperatures during the glow have been recorded simultaneously. Spectra were obtained with a rapid-scanning spectrophotometer using a liquid-air-cooled photomultiplier in conjunction with a cathode-ray oscilloscope. Peaks of the same wavelength were found to be repeated several times in the temperature range of 80-600°K.

Thermal pretreatment was found to enhance the various glow peaks by factors of a few thousands. On prolonged heat-treatment the peaks above room-temperature were found to decrease in intensity, while those at lower temperatures continue to grow even after 80 hours of heat-treatment at 550°C.

Essentially the same results were obtained for both natural (Dead Sea) crystals and artificial ones.

RELATIVE MEASUREMENT OF THE PHOTODETACHMENT CROSS SECTION FOR H^-. Stephen J. Smith and David S. Burch,* Atomic Physics Section, National Bureau of Standards, Washington, D. C. (Received July 13, 1959).

The spectral dependence of the photodetachment cross section for the negative ion of atomic hydrogen has been measured in the range from 4000 A to 13 000 A with approximately 300 A resolution. Measurements were made with twenty-five band-pass filters, each measurement taken relative to the value obtained with a control filter at 5280 A. A probable error of about 2 percent is attached to the relative value obtained for each filter. The results are in significant disagreement with available calculated cross sections.

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FORMATION OF POSITRONIUM IN AN ELECTRON GAS. Joseph Callaway, Department of Mathematics, Queen Mary College, University of London, London, England (Received July 13, 1959).

It is shown to be unlikely that a positronium atom, described by a hydrogenic wave function, can exist in an electron gas of a density corresponding to that found in metals.

ELASTIC SCATTERING OF O^{16} FROM NUCLEI. J. A. McIntyre, S. D. Baker, and T. L. Watts, Yale University, New Haven, Connecticut (Received July 1, 1959).

O^{16} nuclei have been elastically scattered from Au^{197}, Ni (natural isotope abundance), Al^{27}, and C^{12} at a laboratory energy of 158 Mev. The angular distributions obtained show features similar to those obtained for alpha particles scattered from various nuclei. However, diffraction effects are evident for target nuclei of higher atomic number Z with the alpha particle as the bombarding nucleus than with O^{16} as the bombarding nucleus. A survey of the literature is presented to show that diffraction effects become evident for $\eta \lesssim 5$, where $\eta \equiv ZZ'e^2/\hbar v$.

NUCLEAR ENERGY LEVELS IN C^{13}, N^{13}, N^{14}, N^{15}, O^{15}, AND F^{18} FROM He^3-INDUCED REACTIONS. T. E. Young,* G. C. Phillips, and R. R. Spencer,† The Rice Institute, Houston, Texas (Received April 9, 1959).

The Rice Institute 5.5-Mev Van de Graaff accelerator and a magnetic spectrograph have been used to study reaction products produced by He^3 bombardment of B^{11}, C^{12}, C^{13}, N^{14}, and O^{16}. Q values of 9.5, 9.3, 6.31, 5.68, 5.63, 5.49, 4.31, 3.67, and 3.29 Mev were observed for the $B^{11}(He^3, p)C^{13}$ reaction, corresponding to excitations of from 3.7 to 9.90 Mev in C^{13}. The energy levels at 7.69 and 8.87 Mev in C^{13}, for which the Q values were 5.49 and 4.31 Mev, were determined to have widths of approximately 75 and 175 kev, respectively. For the $C^{13}(He^3, p)N^{15}$ reaction the observed Q values were 5.38, 4.33, 3.50, 3.36, 3.09, 2.35, 2.09, 1.61, and 1.50 Mev. The $O^{16}(He^3, p)F^{18}$ reaction revealed the existence of twelve states below 3.9 Mev of excitation in F^{18}. These states occur at 0.943, 1.047, 1.089, 1.128, 1.708, 2.102, 2.521, 3.058, 3.130, 3.355, 3.724, and 3.843 Mev of excitation. The ground-state Q values of the $O^{16}(He^3, \alpha)O^{15}$ and $N^{14}(He^3, d)O^{15}$ reactions were determined to be 4.91 and 1.80 Mev, respectively, and the first excited state of O^{15} was observed at 5.17 Mev by means of the $O^{16}(He^3, p)O^{15}$ reaction.

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THREE-BODY BREAKUP; DEUTERON DISSOCIATION CROSS SECTIONS. James E. Young, University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico (Received September 16, 1957; revised manuscript received August 17, 1959).

An analysis of the reaction $X(d, np)$ has been carried out within the framework of the algebraic theory of scattering. It is possible to cast the transition matrix element governing the process into several forms. In particular, one of these, the asymmetric form, involves no nuclear interactions for one of the deuteron constituents. The other forms, so-called symmetric, contain interactions for all particle pairs and are shown to be inherently superior to the asymmetric expression.

Calculations of the neutron spectrum, at zero degrees in the center-of-mass system, for 6.3-Mev deuterons on deuterons are presented. These are done in an S-wave approximation to illuminate the theoretical discussions; and a comparison is made with experimental data.

It is concluded that the theory as presented is capable of describing the dissociation reaction. Moreover, in its final form, the description involves only those interactions to which we have ready access through experiment.

CHARGE SPECTROMETRY FOR Kr^{79}-Br^{79}. Arthur H. Snell, F. Pleasonton, and John L. Need, Oak Ridge National Laboratory, Oak Ridge, Tennessee (Received July 1, 1959).

The distribution in charge of the Br^{79} atoms formed in the complex e-capture and β^+ decay of radioactive Kr^{79} has been found to consist of two components; one to be mostly associated with the β^+ transitions, and the other with the e-capture transitions. Because of a difference in instru-

mental efficiency, the relative intensity of the two components is undetermined in the present work, but if the value of 9.3 is assumed for the total e-capture/β^+ ratio, then the spectrum is as follows for the successive charges -1, 0, +1, ... +13 (figures in percent): 7.7, 3.7 (estimated), 4.0, 4.7, 12.7, 16.0, 14.3, 13.6, 11.3, 7.7, 3.3, 0.68, 0.13, 0.054, and 0.014.

REACTIONS OF Cu^{63} AND Cu^{65} WITH ALPHA PARTICLES. Norbert T. Porile and David L. Morrison,* Chemistry Department, Brookhaven National Laboratory, Upton, New York (Received July 10, 1959).

Excitation functions have been measured for the (α,n), $(\alpha,2n)$, and $(\alpha,\alpha n)$ reactions on Cu^{63} and Cu^{65}, as well as for the $Cu^{63}(\alpha,pn)$, $Cu^{65}(\alpha,2p)$, and $Cu^{65}(\alpha,2\alpha)$ reactions, for incident alpha particles of 15-41 Mev. The excitation functions for the (α,n), $(\alpha,2n)$, and (α,pn) reactions go through much sharper maxima than the excitation functions for the $(\alpha,\alpha n)$ reactions. Cross sections for the $(\alpha,2p)$ and $(\alpha,2\alpha)$ reactions increase monotonically with bombarding energy and attain values of 2.7 and 2.1 mb at 40 Mev, respectively. The value of $\sigma(\alpha,pn)/\sigma(\alpha,2n)$ for Cu^{63} in the region of maximum yield is 3.3. The maximum cross sections measured for the $(\alpha,\alpha n)$ reactions are 205 mb and 143 mb for Cu^{63} and Cu^{65}, respectively. The effects on the observed cross sections of neutron and proton binding energy differences and of level density differences in the residual nuclei have been considered. The effect of these factors is in accord with the predictions of the statistical theory for the (α,n) and $(\alpha,2n)$ reactions but not for the $(\alpha,\alpha n)$ reaction.

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SCATTERING OF 18-Mev ALPHA PARTICLES BY C^{12}, O^{16}, AND S^{32}. J. C. Corelli,* E. Bleuler, and D. J. Tendam, Department of Physics, Purdue University, Lafayette, Indiana (Received July 15, 1959).

The scattering of 18-Mev alpha particles by gaseous C_3H_8, O_2, and H_2S targets was studied with a multiplate scattering chamber. The elastic angular distributions exhibit the diffraction-like pattern typical of light elements. Carbon and oxygen show a sharp rise above the Rutherford cross sections at the backward angles, with values σ/σ_R of 660 for carbon and 350 for oxygen near 173°. A good fit to the angular distribution for inelastic scattering leading to the first excited state of C^{12} (4.43 Mev, 2^+) is obtained using a $[j_2(qR)]^2$ dependence with $R = 5.5\times10^{-13}$ cm. No direct-interaction analysis is possible for the alpha-particle groups corresponding to the 7.65-Mev and 9.61-Mev levels in C^{12} and to the excited states of O^{16} up to the 8.87-Mev level. All these distributions show strong forward peaking. In the case of inelastic scattering by S^{32} ($Q = -2.24$ Mev), an interaction radius of 6.5×10^{-13} cm can be deduced from the angular distribution, though the agreement with $[j_2(qR)]^2$ is rather poor. A summary of elastic scattering results for elements in the range from $Z = 6$ to $Z = 47$ is presented. Expressions for the second-order geometry and the multiple-scattering corrections are given.

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DETERMINATION OF THE LOW-ENERGY REGION OF THE TRITIUM BETA SPECTRUM. D. C. Conway and W. H. Johnston,* Department of Chemistry, Purdue University, Lafayette, Indiana (Received July 8, 1959).

The tritium beta spectrum has been determined in a proportional-counter spectrometer fitted with field-adjusting electrodes down to an energy of 200 ev. Below 1.2 kev the experimental spectra exhibit positive deviation from the theoretical spectrum which was corrected for the screening effect and for counter resolution. Various possible explanations for the deviation, which amounts to +6.5% at 0.3 kev, are examined. A possible explanation is that w, the energy to produce an ion pair in the counting gas, increases by several percent in the energy interval 1.2 to 0.25 kev.

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NEUTRINO DETECTION BY RESONANCE ABSORPTION IN CRYSTALS AT LOW TEMPERATURES. William M. Visscher, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received July 2, 1959).

An experiment is proposed wherein neutrinos emitted during electron capture by radioactive

impurities in one part of a crystal may be resonantly absorbed by atoms in another part. Essential to the argument is the fact, recently experimentally proved by Mössbauer for gamma rays, that in an emission or absorption by a nucleus bound in a crystal at low temperature the recoil energy is often taken up by the crystal as a whole, with no phonons emitted or absorbed. Thus no energy is lost to recoil, and a neutrino emitted in a monoenergetic process has exactly the right energy to be resonantly absorbed in the inverse process. The feasibility of the experiment depends on the degree to which the electronic impurity levels are broadened relative to the natural width of the neutrino line.

BETA DECAY OF Cd^{115m}. O. E. Johnson and W. G. Smith, Physics Department, Purdue University, Lafayette, Indiana (Received June 22, 1959).

The decay of Cd^{115m} was studied by using NaI(Tl) scintillation counters and a 4π beta scintillation spectrometer. The following gamma rays were observed: 0.485 ± 0.007, 0.935 ± 0.014, 1.14 ± 0.017, and 1.29 ± 0.019 Mev. The 0.485- and 0.935-Mev gammas were found to be in coincidence. There were no gammas in coincidence with the 1.29-Mev gamma. The end-point energy of the ground-state beta transition was determined to be 1.631 ± 0.016 Mev. The shapes and end-point energies of the beta groups in coincidence with the 0.935- and 1.29-Mev gammas were measured. The former has a $\Delta I = 2$, yes, character; and the latter appears to have an allowed or statistical shape: the end-point energies are 0.687 ± 0.008 Mev and 0.335 ± 0.010 Mev, respectively.

HARMONIC OSCILLATOR WAVE FUNCTIONS IN NUCLEAR SPECTROSCOPY. R. D. Lawson,* The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois, and M. Goeppert-Mayer, Argonne National Laboratory, Lemont, Illinois, and Enrico Fermi Institute for Nuclear Studies, Chicago, Illinois (Received July 1, 1959).

Pairing energy calculations are generally carried out assuming that the nucleon wave functions are those of a harmonic oscillator. The two-body interaction is assumed to be some function of σ_1, σ_2, and $\vec{r}_1 - \vec{r}_2$. Since the coordinates appear only in the form $\vec{r}_1 - \vec{r}_2$, it is convenient to write the two-body wave function in terms of the relative coordinate, $\vec{r} = \vec{r}_1 - \vec{r}_2$, and the center-of-mass coordinate, $2\vec{R} = \vec{r}_1 + \vec{r}_2$. The eigenfunction in the new coordinates can be determined by noting that if the two particles are in the same oscillator level, then $\psi(\vec{r}_1, \vec{r}_2)$, which is an eigenfunction of $H_1 + H_2$, is also an eigenfunction of H_1 alone. Transforming H_1 to relative and center-of-mass coordinates implies that the operator $\vec{p} \cdot \vec{P} + mk\vec{r} \cdot \vec{R}$ (where m is the mass of the particle and k is the spring constant of the oscillator) must give zero when operating on the wave function. This condition plus certain requirements arising from the radial form of the oscillator eigenfunctions are sufficient to determine the wave function in the new coordinate system.

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IONIZATION YIELDS FOR FISSION FRAGMENTS. N. G. Utterback* and G. H. Miller,* Institute for Atomic Research and Department of Physics, Iowa State University of Science and Technology, Ames, Iowa (Received June 24, 1959).

The ratio of fission-fragment to alpha-particle ionization produced in total stopping has been measured in the gases helium, argon, and a mixture of helium plus 0.25% argon. A parallel-plate gridded electron collection chamber was used. Gas purity and electron loss were checked by making measurements of absolute average energy per ion pair for alpha particles. These agreed with the values reported for total ion collection. The difference in ionization defects between helium and argon is consistent with the value predicted by Knipp and Ling on the basis of the greater importance of elastic atomic collisions in argon. Because adding a small amount of argon to helium enables the metastable helium atoms to produce ionization, the difference in the fission-fragment to alpha-particle ionization ratios between helium and the helium-plus-argon mixture shows that the ionization-to-excitation ratio in helium is lower for fission fragments than for alpha particles. Arguments are given which indicate that very little "defect" is present for fission fragments in the helium-plus-argon mixture.

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VIOLATION OF THE ISOTOPIC SPIN SELECTION RULE IN THE $Ca^{40}(d,\alpha)K^{38}$ REACTION. Y. Hashimoto* and W. Parker Alford, University of Rochester, Rochester, New York (Received June 18, 1959).

Alpha particles leading to the ground state and first excited state of K^{38} in the $Ca^{40}(d,\alpha)$ reaction have been observed and the excitation energy of the first excited state of K^{38} has been determined as 123 ± 8 kev. The relative intensities of the groups have been measured for incident deuteron energies between 3.2 and 4.1 Mev. Over this range the average intensity of the ground-state group is about 10 times that of the group to the first excited state and it is concluded that the latter is the 0^+, $T=1$ analog of the A^{38} ground state.

Even in the absence of isotopic spin selection rules, the alpha transition to the 0^+ state is expected to be inhibited appreciably by selection rules on angular momentum and parity. A calculation using the statistical model of the nucleus indicates that these decrease the cross section for the $0^+ \rightarrow 0^+$ transition by a factor of about 5 relative to that for the ground-state transition. It thus appears that the isotopic spin selection rule inhibits the transition only by a factor of about two. This breakdown of the isotopic spin selection rule can probably be attributed to Coulomb effects in the compound state. The reduction of the cross section as a result of angular momentum selection rule is probably equally important in other (d,α) reactions that have so far been used to study isotopic spin selection rules. It appears that further experimental results are needed for an understanding of the "isotopic spin forbidden" (d,α) reactions on light nuclei.

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ANGULAR CORRELATION OF Cd^{114} GAMMA RAYS IN SINGLE CRYSTALS OF INDIUM. Arthur G. Duneer, Jr.,* Rensselaer Polytechnic Institute, Troy, New York (Received June 16, 1959).

The measured values of the angular correlation function at $\theta = 150°$ were consistent with the statistical deviation of the mean of ± 0.005 for six measurements with different azimuthal angles between the c axis of an indium metal crystal and the direction of the fixed counter. The electric field gradient at the position of the decaying Cd^{114} nucleus was calculated using a Fermi-Thomas type calculation with account being taken of the antishielding effect of the atomic core electrons.

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LOW-LYING ENERGY LEVELS IN Sc^{41}. H. S. Plendl* and F. E. Steigert, Yale University, New Haven, Connecticut (Received July 9, 1959).

The reaction $Ca^{40}(d,n)Sc^{41}$ was studied at 4.15-Mev bombarding energy using nuclear emulsions as detectors. Four groups of neutrons were observed with Q values of -0.57, -2.43, -2.64, and -2.85 Mev. The observed angular distributions can be fitted with distribution curves obtained from stripping analysis on the basis of $r_0 = 6.0 \times 10^{-13}$ cm and $l_p = 3, 1, 1, 1$, respectively. Two additional groups of questionable assignment were observed at $Q = -1.13$ and -1.41 Mev. Neutron groups from reactions on C^{12} and O^{16} were also observed and served to confirm the beam calibration as well as background and other correction methods.

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CONVERSION ELECTRON SPECTRUM OF Np^{239} AND LEVEL SCHEME OF Pu^{239}. G. T. Ewan, J. S. Geiger, R. L. Graham, and D. R. MacKenzie,* Physics Division, Atomic Energy of Canada Limited, Chalk River, Ontario, Canada (Received May 22, 1959).

The conversion electron spectrum of Np^{239} has been studied at resolution settings of 0.1% and 0.05% with a 100-cm radius air-cored $\pi\sqrt{2}$ β-ray spectrometer, using a proportional counter as detector. One hundred and forty-three lines were identified corresponding to Auger transitions or the internal conversion of γ-rays of energies 44.65, 49.41, 57.26, 61.46, 67.86, 88.06, 106.14, 106.47, 181.7, 209.8, 226.4, 228.2, 254.4, 272.9, 277.6, 285.5, 315.9, and 334.3 kev. The results require the addition of a level at 163.75 kev to the basic level scheme for Pu^{239} proposed by Hollander et al. in order to account for three of the weak gamma transitions observed in this work. The relative conversion line intensities are used to determine the transition multipolari-

ties, mixing ratios, and expected quantum intensities assuming the K- and L-shell theoretical conversion coefficients of Sliv. The precisely determined level spacings and the relative transition probabilities are compared in some detail with those expected from an interpretation of the Pu^{239} level scheme in terms of the "unified model."

New and more accurate K, L, M, and N electron subshell binding energies for Pu have also been deduced from the experimental data of this work. These values are 20 to 100 ev higher than the values tabulated by Hill, Church, and Mihelich, which were estimated by extrapolation from data at lower Z.

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TOTAL NEUTRON CROSS SECTION OF XENON AND KRYPTON. D. P. Mann* and W. W. Watson, Yale University, New Haven, Connecticut, and R. E. Chrien, R. L. Zimmerman,† and R. B. Schwartz,‡ Brookhaven National Laboratory, Upton, New York (Received July 6, 1959).

By the thermal diffusion method, using 12 meters of hot-wire columns, concentrations of the xenon isotopes in good quantity (about 57% Xe^{129} in the light fraction and 27% Xe^{136} in the heavy one) have been produced without recycling. A fair-sized sample of "light" krypton, analyzing better than 50% Kr^{80}, was available from earlier thermal diffusion separation with recycling by Blais and Watson. These samples together with the normal gases were concentrated to thicknesses of about 3.3 g/cm^2 in special gas target holders for use with the Brookhaven fast chopper. For krypton, neutron widths and isotopic identifications have been determined for the following levels: 27.9 ev in Kr^{83}, 39.8 ev in Kr^{82}, 106 ev in Kr^{80}, 233 ev in Kr^{83}, 519 ev in Kr^{84}, 580 ev in Kr^{84}, and 640 ev in either Kr^{78} or Kr^{80}. Total widths and radiation widths have been obtained for the 27.9- and 106-ev levels by thick-thin area analysis. For xenon new resonances are observed at 5.2 ev in Xe^{134}, 9.5 ev in Xe^{129}, 14.1, 46.0, and 76.0 ev all in Xe^{131}, 92.0 ev in Xe^{129}, and 126 ev to be assigned to either the 128, 129, or 130 isotope.

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GENERALIZED ATOMIC MASS LAW. Forrest S. Mozer, Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California, and Lockheed Missiles and Space Division, Palo Alto, California (Received June 29, 1959).

Least-squares analyses have been performed on a set of atomic masses using standard and generalized semiempirical mass laws. Presumably because of errors in the assumed form of the standard mass law, its least-squares coefficients can be determined at best to an accuracy of about 10%, and masses are predicted with an uncertainty of several Mev/c^2. The standard mass law has been generalized by addition of shell-effect and deformation terms. While the least-squares fitting of the generalized mass law is better than for the standard mass law, it is still not possible to predict atomic masses to an accuracy better than a few Mev/c^2. The nuclear deformations and the well depth of the nuclear interaction obtained from the additional mass-law terms are in reasonable agreement with more accurate determinations by other methods. A similar statement applies to the nuclear radius constant as obtained from the least-squares coefficient of the Coulomb energy term. A study has also been made of the effects of additional terms proportional to the absolute value of the isotopic spin, exchange and surface corrections to the Coulomb energy, and the surface correction to the normal isotopic term.

POSITIVE-PION PRODUCTION BY POLARIZED BREMSSTRAHLUNG. R. E. Taylor* and R. F. Mozley, High-Energy Physics Laboratory, Stanford University, Stanford, California (Received June 8, 1959).

By selecting bremsstrahlung produced in a 0.003-in. aluminum radiator at a small angle from the original electron direction, a beam of polarized bremsstrahlung has been obtained from the Stanford linear accelerator. The variation of the polarization and intensity with angle has been studied and compared with theoretical predictions. The polarized beam has been used to study π^+-meson production at 90° c.m. angle and photon energies of 242, 296, 337, and 376 Mev. The ratio of meson production along and at right angles to the electric field vector has

been measured and compared with the values predicted by the relativistic dispersion relation.

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SCATTERING OF NEGATIVE MESONS BY HYDROGEN AT 130 AND 152 Mev. U. E. Kruse* and R. C. Arnold, The Enrico Fermi Institute of Nuclear Studies, and The Department of Physics, The University of Chicago, Chicago, Illinois (Received June 17, 1959).

The elastic scattering and total cross sections for negative π mesons on hydrogen have been measured at 130 and 152 Mev. At both energies, the number of electrons arising from the charge exchange scattering have been determined with a Čerenkov counter. At 152 Mev, recoil protons were counted, and were distinguished from π mesons by energy loss in a scintillator. The real part of the forward scattering amplitude has been determined to be 0.243 ± 0.015 and 0.218 ± 0.016, in units of the meson Compton wavelength, at 130 and 152 Mev, respectively. These values agree within limits of statistical error with the predictions from dispersion relations.

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HIGH-ALTITUDE NEUTRON INTENSITY DIURNAL VARIATION. Robert C. Haymes, New York University, University Heights, New York, New York (Received July 14, 1959).

Two balloon flights in which boron trifluoride neutron counters were carried aloft were launched from Brownwood, Texas, during September, 1958. The flights attained altitudes of 86 000 and 79 000 feet at a conventional geomagnetic latitude of 41°N. They showed that the slow-neutron intensities in the atmosphere had decreased by about 12% since the time of minimum solar activity in 1954. They also show that this decrease was mainly in the low-energy end of the spectrum, as the mean free path for absorption had increased from 180 ± 25 g/cm^2 to 240 ± 30 g/cm^2. A high-altitude decrease apparently associated with the geomagnetic storm of September 25, 1958 was also detected. After achieving altitude, the balloons floated at a constant elevation through sunset. A sharp peak in the intensity which occurs just before sunset at balloon altitudes was detected on both flights. The origin of this phenomenon, which results in a doubling of the intensity for about 25 minutes, is unexplained, although some possible mechanisms are discussed.

SCATTERING OF μ MESONS FROM LEAD NUCLEI. B. Chidley, P. Coldstein,* G. Hinman, R. Summers, and R. Adler, Department of Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 22, 1959).

The angular distribution of 23-Mev μ^- mesons scattered by lead nuclei has been measured using a counter arrangement and also using a propane bubble chamber. The results agree, to the accuracy of the experiments, with the distribution predicted by the ordinary Coulomb interaction of the μ meson and the lead nucleus.

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APPLICATION OF SUM RULES TO ELECTRON-DEUTERON SCATTERING. J. I. Friedman, Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California (Received July 13, 1959).

A measurement of elastic and inelastic electron-deuteron scattering has been made at a momentum transfer of 206 Mev/c and an electron energy of 175 Mev for comparison with the Drell-Schwartz sum rules. The measured value of the non-energy-weighted sum rule is in good agreement with theory. The experimental result for the energy-weighted sum rule is 30% larger than the value given for a pure Wigner potential and is consistent with a Rosenfeld two-body interaction if the analysis is restricted to central forces. The analysis is extended to include tensor forces for comparison with the prediction of the Gartenhaus potential. It is found that the gauge terms introduce major ambiguities when tensor forces are included.

FORM FACTORS IN $K_{\mu 3}$ AND K_{e3} DECAY. S. W. MacDowell,* Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received June 19, 1959).

An investigation of the form factors in $K_{\mu 3}$ and K_{e3} decay amplitudes, on the assumption of local

production of the fermion pair, is carried out by means of dispersion relations techniques. The approximation is made of taking explicit account of K-π scattering in the unitarity condition and all the other contributions are incorporated into a single arbitrary parameter. The validity of this approximation is discussed. The results are expressed in terms of S and P phase shifts for K-π scattering.

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DYNAMICAL STRUCTURE AND DEFINITION OF ENERGY IN GENERAL RELATIVITY. R. Arnowitt, Department of Physics, Syracuse University, Syracuse, New York, S. Deser, Department of Physics, Brandeis University, Waltham, Massachusetts, and C. W. Misner, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received July 6, 1959).

The problem of the dynamical structure and definition of energy for the classical general theory of relativity is considered on a formal level. As in a previous paper, the technique used is the Schwinger action principle. Starting with the full Einstein Lagrangian in first order Palatini form, an action integral is derived in which the algebraic constraint variables have been eliminated. This action possesses a "Hamiltonian" density which, however, vanishes due to the differential constraints. If the differentials constraints are then substituted into the action, the true, nonvanishing Hamiltonian of the theory emerges. From an analysis of the equations of motion and the constraint equations the two pairs of dynamical variables which represent the two independent degrees of freedom of the gravitational field are explicitly exhibited. Four other variables remain in theory; these may be arbitrarily specified, any such specification representing a choice of coordinate frame. It is shown that it is possible to obtain truly canonical pairs of variables in terms of the dynamical and arbitrary variables. Thus a statement of the dynamics is only meaningful after a set of coordinate conditions have been chosen. In general, the true Hamiltonian will be time dependent even for an isolated gravitational field. There thus arises the notion of a preferred coordinate frame, i.e., that frame in which the Hamiltonian is conserved. In this special frame, on physical grounds, the Hamiltonian may be taken to define the energy of the field. In these respects the situation in general relativity is analogous to the parametric form of Hamilton's principle in particle mechanics.

INTEGRAL REPRESENTATIONS OF TWO-POINT FUNCTIONS. S. Deser, Physics Department, Brandeis University, Waltham, Massachusetts, and W. Gilbert and E. C. G. Sudarshan,* Lyman Laboratory, Harvard University, Cambridge, Massachusetts (Received July 15, 1959).

An integral representation for a general matrix element of two field operators between eigenstates labelled by an arbitrary number of momenta is presented. This representation is conveniently parametrized over a set of invariants related to the total momentum and momentum transfer, and explicitly incorporates spectral conditions. Physically interesting dispersion relations for off-forward scattering cannot be obtained from this representation alone.

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COMMUTATION RULES AND SPURIOUS EIGENSTATES IN SOLUBLE FIELD THEORIES. R. E. Norton and W. K. R. Watson, Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, California (Received April 13, 1959; revised manuscript received August 31, 1959).

A study is made of the exactly soluble field theories which are characterized by Hamiltonians quadratic in the field variables. As an example of such a theory, a model describing the electric dipole interaction of photons and a nonrelativistic, harmonically bound electron is studied explicitly. For the cases when the cutoff is sufficiently large to admit the "runaway solutions," it is necessary in order to obtain consistency and a sensible physical interpretation that the theory be reformulated by a unique modification of the equal-time commutation rules of the field operators. The problems which arise here in connection with the "runaway solutions" are closely related to the troubles of ghost states and negative transition "probabilities" which have been demonstrated or suggested to exist in other theories. It is hoped, therefore, that the procedure of reformulation required here may be a

guide for the eventual resolution of the ghost state problem should it be demonstrated that such problems actually exist in the physical, relativistic field theories.

RIGID FRAMES IN RELATIVITY. Ezra T. Newman and Allen I. Janis, University of Pittsburgh, Pittsburgh, Pennsylvania (Received July 13, 1959).

A new approach is presented for defining and obtaining rigid frames of reference. The results are shown to be equivalent to those of Rosen. The advantage of the present approach is that exact solutions can be obtained in certain simple cases, as well as approximate solutions in general.

STABILITY OF NUCLEAR MATTER. K. Sawada* and R. M. Rockmore, Brookhaven National Laboratory, Upton, New York (Received July 16, 1959).

The stability condition for a model of collective excitations of nuclear matter due to Glassgold, Heckrotte, and Watson is derived; it is shown that it may be satisfied for the actual parameters (nuclear radius, strength of potential, etc.). It is possible that nuclear matter is stable in their approximation, although there is no compressional collective mode. The stability condition seems to imply values of η, the radius parameter used by Karplus and Watson, equal to or smaller than those values obtained from their simple saturating nuclear model.

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NUCLEON STRUCTURAL CORRECTIONS TO FIRST FORBIDDEN UNIQUE BETA TRANSITIONS. J. F. Dreitlein, Department of Physics, Washington University, St. Louis, Missouri (Received July 13, 1959).

The coupling of leptons to a conserved isovector current is shown to result in a small change in the shape of the electron momentum spectrum in forbidden unique beta transitions. The experimental advantages and disadvantages of studying such nucleon structural effects in forbidden unique transitions rather than in allowed transitions, as suggested by Gell-Mann, are discussed. To facilitate the computations, the beta decay interaction Hamiltonian is written in a form in which the terms leading to the "nucleon structural" corrections associated with the coupling of the leptons to a conserved isovector current and the terms leading to effects of comparable order of magnitude are simply identified.